The Bell System Technical Journal

Devoted to the Scientific and Engineering Aspects of Electrical Communication

Published quarterly by the American Telephone and Telegraph Company, through its Information Department, in behalf of the Western Electric Company and the Associated Companies of the Bell System

Address all correspondence to the Editor
Information Department
AMERICAN TELEPHONE AND TELEGRAPH COMPANY
195 BROADWAY, NEW YORK, N. Y.

50c. Per Copy   *$1.50 Per Year*

Vol. I   JULY, 1922   No. 1

FOREWORD

MODERN industry is characterized by the extent to which scientific research and technique based on precise study have contributed to its progress. So complete has been the adaptation of and reliance on scientific research in many industries that it is difficult at this time to visualize the state of affairs of two or three decades ago, when substantially all industry on its technical side was dependent for advancement on cut-and-try, rule-of-thumb, methods of development. Today in many industries the management would not think of embarking on a new project without consulting their research engineers.

Many industries have proved the benefits to be derived from the utilization of that organized knowledge provided both in the fields of the physical sciences and in those newer fields which have to do with psychology and economics. There are still greater numbers of industrial organizations where the adoption of scientific methods has been slow. However, the time will undoubtedly come when every industry will recognize the aid it can derive from scientific research in some form as it now recognizes its dependence for motive power on steam or electricity rather than on muscular activity.

Upwards of one hundred years ago there was adopted in earnest by scientific men, principally in university laboratories, the program of searching deeper into the unknown, to discover new principles and new relationships of a kind which had at the time very little apparent practical interest to mankind as a whole.

Out of this work, and in time, have grown entirely new industries. From the fact that these industries sprang directly from the research laboratory, it was inevitable that they should be conspicuous because of the number of their men trained in the methods of scientific research. Equally inevitable was it that these new fields of endeavor, originating as they did and being staffed as they were, should be the ground where industrial research would find its first and largest development. And not the least of the advantages which obtained in these newer industries was the absence of age-long traditions tending to ultra-conservatism as to new undertakings, and more particularly as to the employment of the new types of mind.

The results up to the present indicate clearly that the electrical and chemical fields in industry as we know them today, are the places where the greatest advances have been made in the utilization of research methods and research men. Other, older and more basic industries are rapidly following the general path marked out by the successes already obtained in these fields. Hence, it is expected that shortly all industrial activities will be based on the results obtained by trained investigators, using the tools of modern scientific investigation.

Just as applied electricity is a leading exemplar of the benefits to be obtained by an intelligent use of scientific knowledge, so electrical communication of intelligence is a leading exemplar in the field of applied electricity. This branch of applied electricity is a pioneer among those recognizing the practical value of scientific research. It is interesting to note that electrical communication is credited with having organized a research laboratory prior to the first university course in electrical engineering.

More than ever before, the communication engineer must seek exact solutions of his problems. If his results do not always attain the certainty he desires, the reason is the absence of complete knowledge with regard to one or more essential facts. But true knowledge of what things limit the solution of a problem is frequently more than half the battle of obtaining the missing facts. Sometimes these unknown facts can be obtained by a search through the remoter parts of the vast scientific storehouses which have been built in times past. Frequently, however, the search discloses the entire absence

of the thing sought for, and new researches are begun with definite ends in view. Thus it has come about that the communication engineer has become an original investigator and is extending the boundaries of human knowledge and supplementing the advances of pure science to find solutions for his various and sundry problems.

Hence, while well equipped physical and chemical laboratories are still a necessary part of the communication engineer's equipment, he is equally active in pushing his investigations in many other directions. Questions involved in the making of proper rate schedules and adequate fundamental plans for new construction are originating profound researches in such fields as political science, psychology and mathematics. A casual examination of recent technical literature dealing with electrical communication would show articles which touch upon almost every branch of human activity, which we designate as science.

With this intense and growing interest in the proper application of scientific methods to the solution of the problems of electrical communication, it is natural that a widespread desire should have arisen for a technical journal to collect, print or reprint, and make readily available the more important articles relating to the field of the communication engineer. These articles are now appearing in some fifteen or twenty periodicals scattered throughout the world and in the majority of instances receive their first and last printing in these widely separated mediums. The need already felt for such a journal will grow keener as new developments extend the scope of the art and the specialization of its engineers of necessity increases. It is hoped that the BELL SYSTEM TECHNICAL JOURNAL will fill this need, and as implied above, it is intended that the range of subjects treated in the JOURNAL will be as broad as the science and technique of electrical communication itself.

While many of the articles which will appear in the JOURNAL will be original presentations of some phase of the research or development or other technical work of the Bell System, it is not intended that the JOURNAL should be the sole means by which this work is presented. Just as in the past, original articles and papers will continue to be presented before various societies and in different technical and non-technical magazines. Moreover, the JOURNAL will reprint articles on important research and development work in the communication field generally so that the results of such work may be given greater publicity and become of greater value to communication engineers.

A New Type of High Power Vacuum Tube

By W. WILSON

SYNOPSIS: The type of vacuum tube described in the present article is likely to become one of the most remarkable devices of modern electrical science. Vacuum tubes capable of handling small amounts of power have been extensively used during the past few years as telephone repeaters and as oscillators, modulators, detectors and amplifiers in radio transmission and other fields. Practically all such tubes have depended upon thermal radiation from the plates to dissipate the electrical energy which the device necessarily absorbs during its operation. With present methods of construction, and using glass for the containing bulb, a fairly definite upper limit can be set for the power which a radiation cooled tube can handle; as the author points out, this limit gives a tube capable of delivering about 1 to 2 k. w. when used as an oscillator.

Contrasted with this, one of the water-cooled vacuum tubes described herewith, although scarcely two feet in length and weighing only ten pounds, is capable of delivering 100 k. w. of high frequency energy. Another tube of similar construction, but somewhat smaller in size, and capable of delivering about 10 k. w. is also described. It is expected that these water-cooled tubes will find important applications in radio telephony and telegraphy.

Although the principle of operation of the water-cooled tube described in this article is identical from an electrical point of view with that of the small tubes which are now so very familiar, their practicability has only been made possible by a new and striking development in the art of sealing metal to glass. In the case of the 100 k. w. tube the seal between the cylindrical copper anode and glass portion is 3.5 inches in diameter.

The remarkable character of these copper-in-glass seals is evidenced by the fact that they do not depend upon a substantial equality between the coefficient of expansion of the metal and glass. To Mr. W. G. Houskeeper of the Bell System Research Laboratory at the Western Electric Company, goes the credit for developing the copper-in-glass seals. As the article brings out, Mr. Houskeeper has also invented means for sealing heavy copper wire and strip through glass in such a way that the best vacua can be maintained under wide changes of temperature.—*Editor.*

THE development of wireless telephony and the use of continuous wave transmission in wireless telegraphy have led to the general adoption of the vacuum tube as the generator of high frequency currents in low power installations.

The ordinary form of vacuum tube is, however, ill suited for the handling of large amounts of power, and at the large wireless stations where the plant is rated in hundreds of kilowatts either the arc or the high frequency alternator is used.

The undoubted advantages to be derived from the use of vacuum tubes, especially in the field of wireless telephony where the output power must be modulated to conform to the intricate vibration pattern of the voice, has led to a demand for tubes capable of handling amounts of power comparable with those in use at the largest stations.

That the development of such tubes was of great importance was recognized by the engineers of the Bell Telephone System in the early days of the vacuum tube art. The experiments at Arlington,

Virginia, in which speech was first transmitted across the Atlantic to Paris and across the Pacific to Honolulu, required the use of nearly 300 of the most powerful tubes then available, each capable of handling about 25 watts, and the difficulties encountered in operating so many tubes in parallel gave added impetus to the development of high power units.

It is the object of the present paper to deal with the various steps in the development of high power tubes as carried out in the Bell System research laboratories at the Western Electric Company.

The usual type of vacuum tube consists of an evacuated glass vessel in which are enclosed three elements, the filament, the plate, and the grid. When the tube is in operation an electron current flows between the filament which is heated by an auxiliary source of power and the plate, the magnitude of this current being controlled by the grid.

The passage of the current through a thermionic tube is accompanied by the dissipation in the plate of an amount of power which is comparable to the power delivered to the output circuit and which manifests itself in the form of heat. This causes the temperature of the plate in the usual type of tube to rise until the rate of loss of heat by radiation is equal to the power dissipated. Some of the heat liberated by the plate is absorbed by the walls of the containing vessel which consequently rise in temperature. These factors, together with a consideration of the size of plate that can be conveniently suspended inside a glass bulb and the size of glass bulb that can be conveniently worked, set a limit of about 1 to 2 k. w. for the power that can be dissipated in the plate of a commercial vacuum tube of this type. The plates are generally constructed of molybdenum or some other refractory metal and the containing vessel made of hard glass.

The use of quartz as the containing vessel offers certain advantages which tend to raise the power limit somewhat and this material has been used for power tube purposes in England.

It is apparent then that in the development of vacuum tubes capable of handling large amounts of power means other than radiation must be used for removing the heat dissipated at the plate, and development of tubes along these lines was undertaken by Dr. E. R. Stoekle and Dr. O. E. Buckley.

Dr. Stoekle had already worked for some years on the problem of removing the heat dissipated at the anode of a thermionic tube by making the anode a part of the outside wall of the vessel and thus making it possible to convey the heat directly away from it by means of circulating water. This was clearly the right principle but as is obvious to those who are familiar with these devices, great difficulties

presented themselves in the mechanical construction of large tubes in which vacuum tight joints must be made and maintained between glass and large masses of metal. The importance of the problem, however, was such that Stoekle and Buckley pushed on in the face of difficulties to the construction of tubes which could handle kilowatts where previous tubes could only handle watts.

A step in the direction of overcoming these difficulties was made by Messrs. Schwerin and Weinhart, who were working with Dr. Buckley on the problem, and who suggested that the anode might be made in the form of a tube or thimble of platinum sealed into a glass vessel and kept cool by passing water through it.

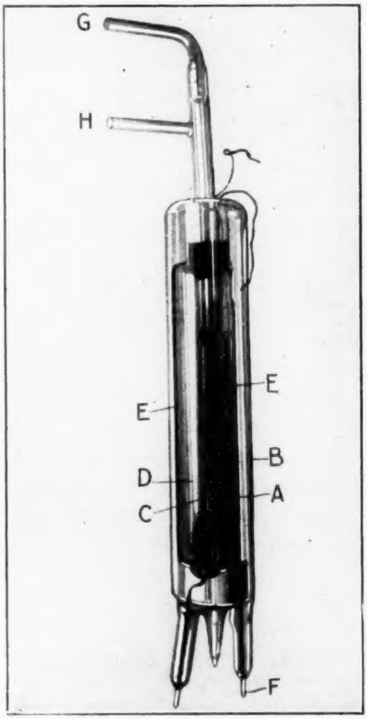


Fig. 1

This suggestion led to the development of a tube which, although not the one finally adopted, is discussed in some detail since it was the first one to be pushed to such a point as to give promise of economical commercial manufacture.

The tube is shown in Fig. 1. The anode consists of a platinum cylinder A, 7″ long and .625″ wide, which is sealed into the center of the glass cylinder B. The end of the platinum cylinder remote from the seal is closed. The anode is surrounded by the grid C and by the filament D, which are supported by the glass arbors E. The current for the filament is led into the tube through the platinum thimbles F.

The anode is kept cool by means of a supply of water passing into the anode through the tube G and leaving by the tube H.

A number of tubes having this general type of construction were made up and it was found possible to dissipate as much as 15 k. w. in the anode.

As soon as the pressure of work more directly connected with the necessities of the war would permit, Mr. W. G. Houskeeper and Dr. M. J. Kelly undertook the further improvement of the water-cooled tube, the former assuming the task of developing the mechanical structure, and the latter that of determining the electrical design and the process of tube exhaust.

Mr. Houskeeper adopted into the construction of the tube a remarkable type of vacuum seal which he had previously developed. These seals are made between glass and metal and can be made in any desired size. They are capable of withstanding repeated heating and cooling over wide ranges of temperature, from that of liquid air to 350° C, without cracking and without impairment of their vacuum holding properties.

It is no exaggeration to say that the invention of these seals has made possible the construction of vacuum tubes, capable of handling in single units, powers of any magnitude which may be called for in wireless telegraph and telephone transmission.

The underlying principle connected with the making of this seal consists in obtaining an intimate connection between the glass and metal, either by chemical combination or by mere wetting, and in so proportioning the glass and metal portions of the seal that the stresses produced when the seal is heated or cooled will not be great enough to rupture either the glass or the junction between the glass and metal.

The three principal types of seals developed by Mr. Houskeeper are known as the ribbon seal, the disc seal and the tube seal.

If a copper ribbon is directly sealed through glass it is found that the glass and copper adhere along the flat faces of the seal but that ruptures occur along the edges as shown in Fig. 2 (a). This is due to the fact that as the seal cools after being made, the glass in contact with metal is capable of resisting the shearing and tensile stresses

that occur along the faces, while the glass wrapping round the edges of the ribbon is called upon to withstand much greater tensile stresses and gives way. If the edges of the ribbon are sharpened as shown in Fig. 2 (b), a tight seal results, the reason being that the forces of

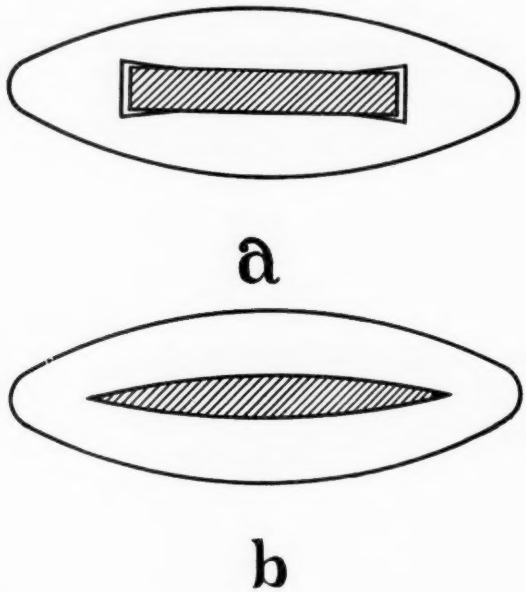


Fig. 2

adhesion between the glass and copper acting along the flat contact faces are sufficient to stretch the thin copper at the edge and prevent its drawing away when cooled. There is a definite relation between the elastic properties of the metal and glass and the angle of edge that can be used for a successful seal.

By proper shaping of the metal ribbon, seals have been successfully made up to very large sizes. Some of these are shown in Fig. 3, the the largest in the photograph being about 1″ in width, and capable of successfully conducting a current of 150 to 200 amperes.

The principles involved in the making of the disc seal are the same as those involved in making the ribbon seal. If a metal disc is sealed wholly into glass the edges must be sharpened or the glass and copper break away from each other as in the case of the ribbon seal.

In the general use to which these seals are put there is no necessity for having the glass surround the circumference of the copper disc

and the necessity for sharpening the edge is obviated by allowing the glass to adhere to the flat portion of the disc only, care being taken to prevent its flowing around the edge. It is necessary to have a ring of glass on both sides of the seal in order to equalize the bending stresses

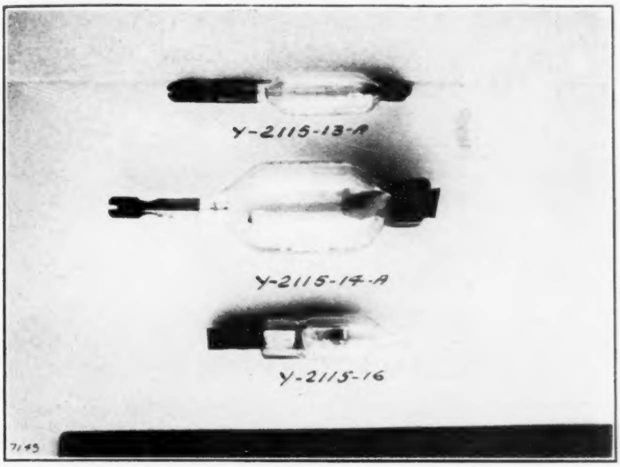


FIG. 3

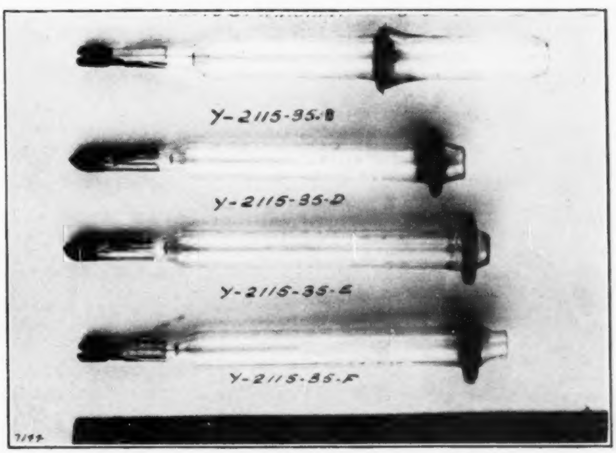


FIG. 4

which would otherwise tend to break the glass and copper away from each other. Successful disc seals have been made with copper up to 1-10″ thick. There is, of course, a certain maximum thickness that can be used for a seal of a given diameter and it is preferable to keep well below this limit.

The seals shown in Fig. 4 close the ends of glass tubes to the other ends of which are sealed pilot lamps for the purpose of testing the vacuum. Tubes sealed in this way have been kept a number of years without any impairment of the vacuum.

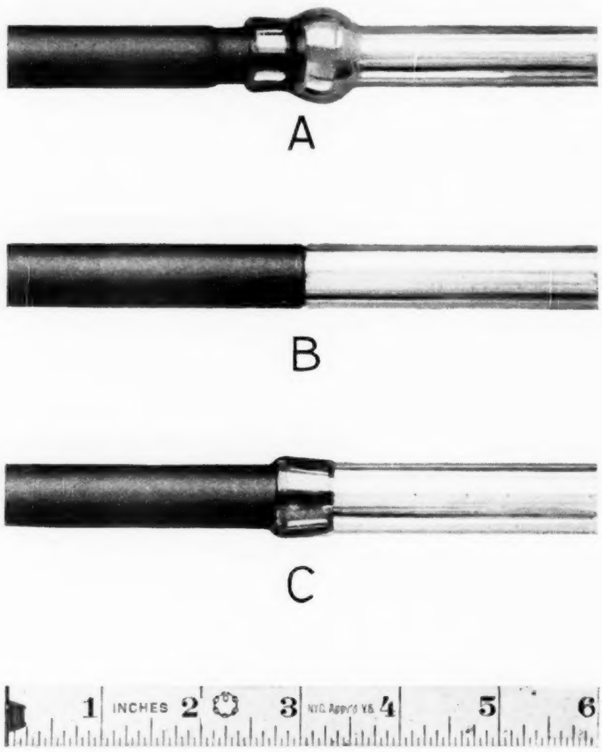


Fig. 5

The third type of seal and the most important in connection with the present problem is the tube seal shown in Fig. 5. This furnishes the means of joining metal and glass tubes end to end and is used in the water-cooled tube to attach the anode to the glass cylinder which

serves to insulate the other tube elements. As in the case of the disc seal, it can be made either with the edge of the metal not in contact with the glass, as shown at A, or with the metal sharpened to a fine edge which is in contact with the glass. The glass may be situated either inside or outside of the metal, see B and C.

The first thermionic tubes in which these seals were embodied were made of copper and were designed to operate at 10,000 volts and to give about 5 k. w. output.

A photograph of one of these tubes is shown in Fig. 6; and the filament grid assembly is shown in Fig. 7.

FIG. 6

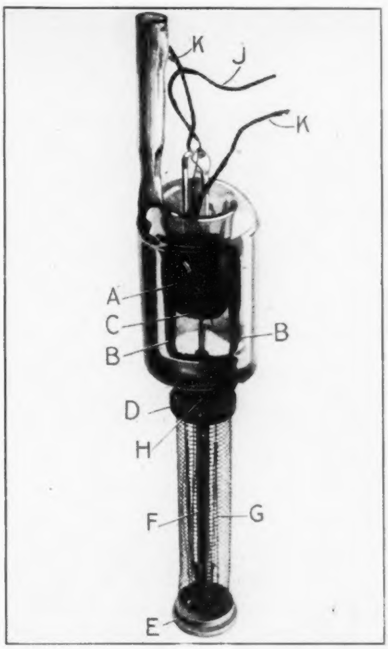


FIG. 7

The anode consists of a copper tube 1.5″ in diameter and 7.5″ long. A copper disc is welded to one end forming a vacuum-tight joint. The other end which is turned down to a knife edge is fused directly to a glass tube.

The filament grid assembly consists of two lavite discs D and E, spaced 5″ apart by a seamless steel tube. The grid F is made in the form of a helix, and is held in position by allowing the ends of the longitudinal wires, to which the turns of the helix are welded, to pass through holes in the lavite blocks D and E. The filament G is mounted between hooks fastened to the lavite blocks and is kept taut by the springs H. The grid lead is shown at J, and the filament leads at K K. In this tube platinum seals are used for the lead wires. The use of the springs H make it necessary to supply the filament with current from the opposite end of the assembly and this is done by passing the current through the steel support tube and returning it through a lead passing through this tube and insulated from it by a quartz tube.

The whole assembly is carried by two supports B B. These supports are welded to a corrugated nickel collar A which grips the glass stem C.

The pumping of these tubes at first presented considerable difficulty, chiefly on account of the large amount of occluded gas contained by the metal parts. This caused the time of pumping of the tube to be very long and a dangerous warping of the internal structure developed owing to the fact that during exhaust the tube elements are maintained at a much higher temperature than they are subjected to during normal operation. The trouble was overcome by heating the various parts of the tube to as high a temperature as possible in a vacuum furnace, prior to the final assembly, and thus getting rid of a large amount of the occluded gases. The anode was preheated before the glass seal was made and the whole filament grid assembly was preheated just before it was mounted on the glass stem. The preheating of the parts brought about an enormous reduction in the time required for pumping and gave a much more uniform product.

Although successful from the standpoint of operation, this tube had several undesirable features that it was thought well to eliminate. In the first place the welding of the end into the tube was not particularly desirable, and in general any troubles that occurred due to leaks in the metal could be traced to this point. Further, in the assembly of the tube there were a very large number of welds to be made which constituted points of weakness at the high temperature necessary for the evacuation of the tubes. It was, therefore, decided to go to a type of tube in which the anode would be drawn in one piece and in which as many welds as possible would be eliminated in the assembly of the internal elements. At the same time it was considered desirable to go to a somewhat larger type of structure in which high

tension insulation could be more easily provided and a larger tube was, therefore, designed capable of delivering 10 k. w. to an antenna at a plate voltage of 10,000 volts.

The final form adopted for this tube is shown in Figs. 8 and 9.

FIG. 8

The anode A is drawn from a piece of sheet copper and is 9″ long and 2″ in diameter. The copper flare B is turned down to a sharp edge and a glass bulb C sealed thereto. The grid and plate assembly is shown at D. The structure is supported by four molybdenum rods, which are threaded and secured by means of nuts to the lavite pieces E and F. The filament is made of 19.5″ of .025 pure tungsten wire purchased from the General Electric Company and is formed and secured to two of the molybdenum rods at G and H. The

power consumed in it during operation is .75 k. w. It is guided by the hooks J. The filament leads are shown at K, K and are led through the glass by the copper disc seals L, L. The grid is a molybdenum helix and is supported by the molybdenum rods M which are fixed to the lavite block E and slide on the outside of the lavite block F. The whole structure is mounted on the flare R by means of the

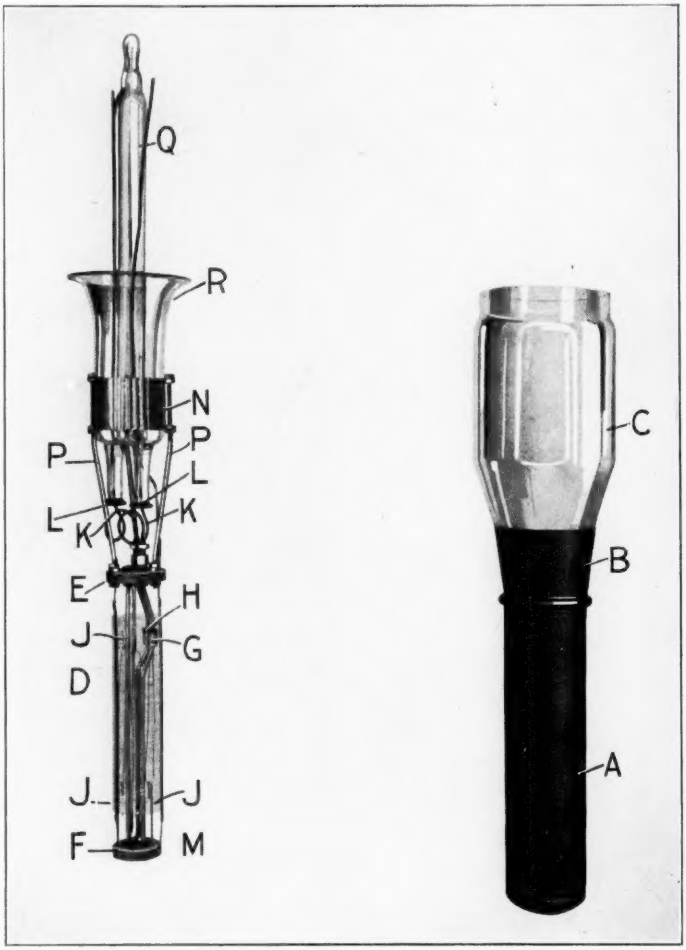


Fig. 9

nickel collar N and the support rods P. The grid lead is brought out through the tube Q. The tube is completed by sealing together the flare R and the bulb C.

In this tube all welds except those in the collar N are eliminated, the assembly being bolted together. The drawing of the anode does

FIG. 10

away with the leaks that were troublesome in the older tubes and the manufacture of the tube can be carried out with certainty.

With this tube as much as 12 k. w. have been obtained in an artificial antenna working at 12,000 volts. This power was obtained at a frequency of 600,000 cycles corresponding to 500 meters wave length. The difficulties of obtaining this amount of power at this frequency using a number of smaller tubes in parallel, are obvious to anyone who is acquainted with the problem. On a D. C. test the anode was found to be capable of dissipating 26 k. w. when cooled with water.

The success which had attended the development of a tube of this high power capacity indicated the possibility of constructing still larger tubes and it was decided to proceed with the development of a tube capable of delivering at least 100 k. w. into an antenna.

The development proceeded with a few minor alterations along the lines of the smaller tube, nominally rated at 10 k. w. and the 100 k. w.

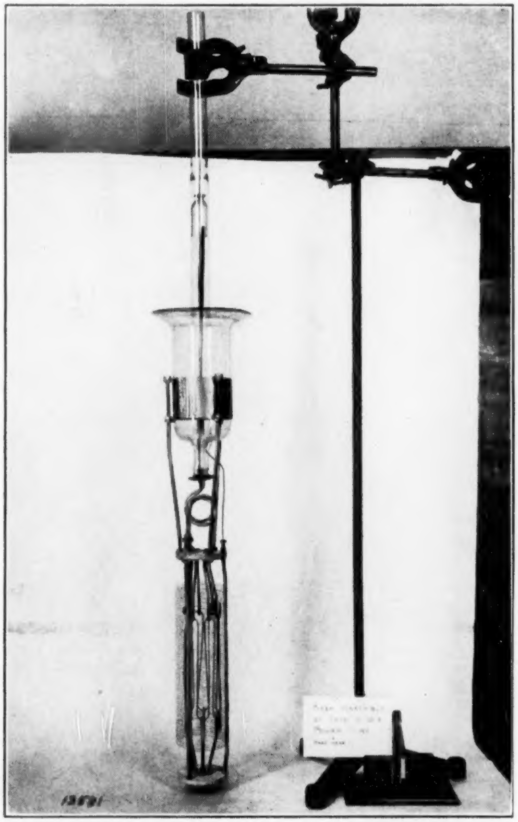


FIG. 11

tube as now developed is shown in Figs. 10 and 11. The anode which is made of a piece of seamless copper tubing closed by a copper disc welded into the end, is 14″ long and 3.5″ in diameter. The filament is of tungsten and is .060″ in diameter and 63.5″ long. The current required to heat it is 91 amperes and the power consumed in it 6 k. w.

The filament leads are of copper rod one eighth of an inch in diameter and are sealed through 1″ copper disc seals. The grid is of molybdenum and is wound around three molybdenum supports.

The handling of the parts of this tube during manufacture presents a task of no mean magnitude and numerous fixtures have been devised to assist in the glass working. It has been found necessary for instance to suspend the anode in gimbals during the making of the tube seal owing to its great weight, and special devices have been made to hold the filament grid assembly in place while it is being sealed in, otherwise the strains produced by its weight cause cracking of the seal.

The significance of this development in the radio art cannot be overestimated. It makes available tubes in units so large that only a very few would be necessary to operate even the largest radio stations now extant, with all the attendant flexibility of operation which accompanies the use of the vacuum tube.

From the standpoint of wireless telephony the development of these high power tubes gives us the possibility of using very much greater amounts of power than have ever been readily available before. The filaments in these tubes have been made so large that the electron emission from them will easily take care of the high peak currents accompanying the transmission of modulated power.

The 100 k. w. tube by no means represents the largest tube made possible by the present development. There is no doubt that if the demand should occur for tubes capable of handling much larger amounts of power they could be constructed along these same lines.

Direct Capacity Measurement[1]

By GEORGE A. CAMPBELL

SYNOPSIS: Direct capacity, direct admittance and direct impedance are defined as the branch constants of the particular direct network which is equivalent to any given electrical system. Typical methods of measuring these direct constants are described with especial reference to direct admittance; the substitution alternating current bridge method, due to Colpitts, is the preferred method, and for this suitable variable capacities and conductances are described, and shielding is recommended. Proposed methods are also described involving the introduction of electron tubes into the measuring set, which will reduce the measurement to a single setting or deflection. This gives an alternating current method which is comparable with Maxwell's single null-setting cyclical charge and discharge method. Special attention is drawn to Maxwell's remarkable method which is entirely ignored by at least most of the modern text-books and handbooks.

THE object of this paper is to emphasize the importance of direct capacity networks; to explain various methods of measuring direct capacities; and to advocate the use of the Colpitts substitution method which has been found preeminently satisfactory under the wide range of conditions arising in the communication field.

About thirty years ago telephone engineers substituted the so-called " mutual capacity " measurement for the established " grounded capacity " measurement; this was a distinct advance, since the transmission efficiency is more closely connected with mutual capacity than with grounded capacity. Mutual capacity, however, can give no information respecting crosstalk, and accordingly, about twenty years ago, I introduced the measurement of " direct capacity " which enabled us to control crosstalk and to determine more completely how telephone circuits will behave under all possible connections.

For making these direct capacity measurements alternating currents of telephone frequencies were introduced so as to determine more exactly the effective value of the capacity in telephonic transmission, and to include the determination of the associated effective direct conductances which immediately assumed great importance upon the introduction of loading.

Telephone cables and other parts of the telephone plant present the problem of measuring capacities which are quite impossible to isolate, but which must be measured, just as they occur, in association with other capacities; and these associated capacities may be much larger than the particular direct capacity which it is neces-

[1] This article is also appearing in the August issue of the *Journal of the Optical Society of America and Review of Scientific Instruments.* An appendix is added here giving proofs of the mathematical results.

sary to accurately measure, and have admittances overwhelmingly larger than the direct conductance, which is often the most important quantity. This is the interesting problem of direct capacity measurement, and distinguishes it from ordinary capacity measurements where isolation of the capacity is secured, or at least assumed.

The substitution alternating current bridge method, suggested to me in 1902 by Mr. E. H. Colpitts as a modification of the potentiometer method, has been in general use by us ever since in all cases where accuracy and ease of manipulation are essential.

After first defining direct capacities and describing various methods for measuring them, this paper will explain how this may all be generalized so as to include both the capacity and conductance components of direct admittances, and the inductance and resistance components of direct impedances.

Definition of Direct Capacity

It is a familiar fact that two condensers of capacities C_1, C_2, when in parallel or in series, are equivalent to a single capacity $(C_1 + C_2)$ or $C_1 C_2/(C_1 + C_2)$, respectively, directly connecting the two terminals. These equivalent capacities it is proposed to call direct capacities. The rules for determining them may be stated in a form having general applicability, as follows:

Rule 1. The direct capacity which is equivalent to capacities in parallel is equal to their sum.

Rule 2. The direct capacity between two terminals, which is equivalent to two capacities connecting these terminals to a concealed branch-point, is equal to the product of the two capacities divided by the total capacity terminating at the concealed branch-point, *i.e.*, its grounded capacity.

These rules may be used to determine the direct capacities of any network of condensers, with any number of accessible terminals and any number of concealed branch-points. Thus, all concealed branch-points may be initially considered to be accessible, and they are then eliminated one after another by applying these two rules; the final result is independent of the order in which the points are taken; all may, in fact, be eliminated simultaneously by means of determinants[2]; a network of capacities, directly connecting the accessible terminals, without concealed branch-points or capacities in parallel, is the final result. Fig. 1 shows the two elementary cases of direct capacities and also, as an illustration of a more complicated system, the bridge

[2] See appendix, section 1, for a discussion of determinant solutions.

circuit, with three corners 1, 2, 3 assumed to be accessible, and the fourth inaccessible, or concealed. Generalizing, we have the following definition:

The direct capacities of an electrical system with n given accessible terminals are defined as the $n(n-1)/2$ capacities which, connected between each pair of terminals, will be the exact equivalent of the system in its external reaction upon any other electrical system with which it is associated only by conductive connections through the accessible terminals.

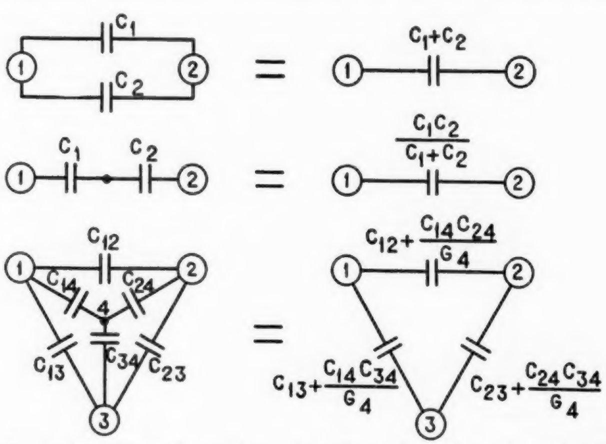


Fig. 1—Equivalent Direct Capacities. $G_4 = C_{14} + C_{24} + C_{34}$ = Grounded Capacity of Branch-Point 4

The total direct capacity between any group of the terminals and all of the remaining accessible terminals, connected together, is called the grounded capacity of the group.

This definition of direct capacity presents the complete set of direct capacities as constituting an exact, symmetrical, realizable physical substitute for the given electrical system for all purposes, including practical applications. Direct capacities are Maxwell's "coefficients of mutual induction," but with the sign reversed, their number being increased so as to include a direct capacity between each pair of terminals.

In considering direct capacities we exclude any direct coupling, either magnetic or electric, from without with the interior of the electrical system, since we have no concern with its internal structure; we are restricted to its accessible, peripheral points or terminals; some care has been taken to emphasize this in the wording of the definition.

Additive Property of Direct Capacities

Connecting a capacity between two terminals adds that capacity to the direct capacity between these terminals, and leaves all other direct capacities unchanged. Connecting the terminals of two distinct electrical systems, in pairs, gives a system in which each direct capacity is the sum of the corresponding two direct capacities in the individual systems. Joining two terminals of a single electrical system to form a single terminal adds together the two direct capacities from the two merged terminals to any third terminal, and leaves all other direct capacities unchanged, with the exception of the direct capacity between the two merged terminals, which becomes a short circuit. Combining the terminals into any number of merged groups leaves the total direct capacity between any pair of groups unchanged, and short-circuits all direct capacities within each group.

These several statements of the additive property of direct capacities show the simple manner in which direct capacities are altered under some of the most important external operations which can be made with an electrical network, and explain, in part, the preeminent convenience of direct capacity networks.

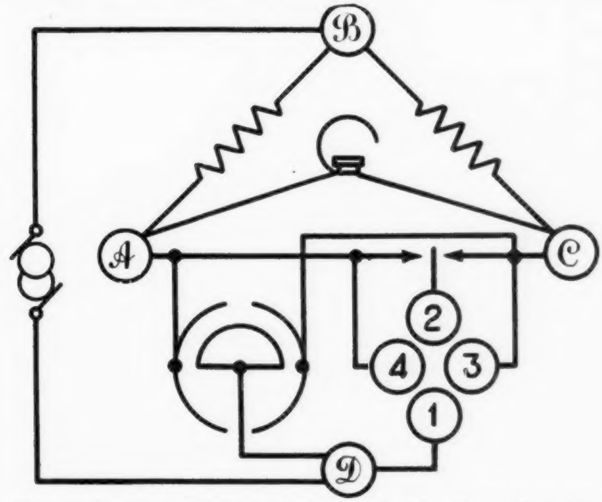


Fig. 2—Colpitts Substitution Bridge Method for Direct Capacity

Since the additive property of direct capacities is sufficient for explaining the different methods of measuring direct capacities we may now, without further general discussion of direct capacities, proceed to the description of the more important methods of measurement.

Colpitts Substitution Bridge Method, Fig. 2

The unknown direct capacity is shifted from one side of the bridge to the other, and the balance is restored by adjusting the capacity standard so as to shift back an equal amount of direct capacity. The method is therefore a substitution method, and the value of the bridge ratio is not involved. Both the standard and the unknown remain in the bridge for both settings, so that the method involves transposition rather than simple, ordinary substitution.

Details of the method as shown by Fig. 2 are as follows: To measure the direct capacity C_{12} between terminals 1 and 2 connect one terminal (1) to corner $\mathcal{D}$ of the bridge, and adjust for a balance with the other terminal (2) on corner $\mathcal{A}$ and then on $\mathcal{C}$, while each and every one of the remaining accessible terminals (3, 4, . . .) of the electrical system is permanently connected during the two adjustments to either corner $\mathcal{A}$ or $\mathcal{C}$. If the direct capacities in the standard condenser between corners $\mathcal{A}$ and $\mathcal{D}$ are C', C'' in the two balances,

$$C_{12} = C'' - C'$$

and if the bridge ratio is unity[3],

$$C_{13} - C_{14} = C' + C'' - 2C^\circ,$$

where C_0 is the standard condenser reading when the bridge alone is balanced.

Two settings are required by this method for an individual direct capacity measurement, but in the systematic measurement of all the direct capacities in a system the total number of settings tends to equal the total number of capacities, when this number becomes large. The number of settings may always be kept equal to the number of capacities by employing an equality bridge ratio, and using the expression for the direct capacity difference given above. The same remarks also hold for the group of direct capacities connecting any one terminal with all the other terminals.

In general, ground is placed upon corner $\mathcal{C}$ of the bridge, but is transferred to corner $\mathcal{D}$, if it is connected to one terminal of the required direct capacity. The arbitrary distribution of the other terminals between corners $\mathcal{A}$ and $\mathcal{C}$ may be used to somewhat control the amount of standard capacity required; or it may be helpful in reducing interference from outside sources, when tests are made upon extended circuits. The grounded capacity of a terminal or group of terminals is measured by connecting the group to $\mathcal{C}$, and all of the remaining terminals together to $\mathcal{D}$.

[3] See appendix, section 2.

The excess of one direct capacity C_{12} over another C_{56} is readily determined by connecting terminals 1 and 5 to corner $\mathcal{D}$, terminals 3, 4, 7, 8, . . . to corner $\mathcal{C}$ or $\mathcal{A}$, and then balance with terminals 2 and 6 on $\mathcal{A}$ and $\mathcal{C}$, respectively, and repeat, with their connections reversed.

Potentiometer Method, Fig. 3

The required direct capacity C_{12} is balanced against one of its associated direct capacities, augmented by a standard direct capacity

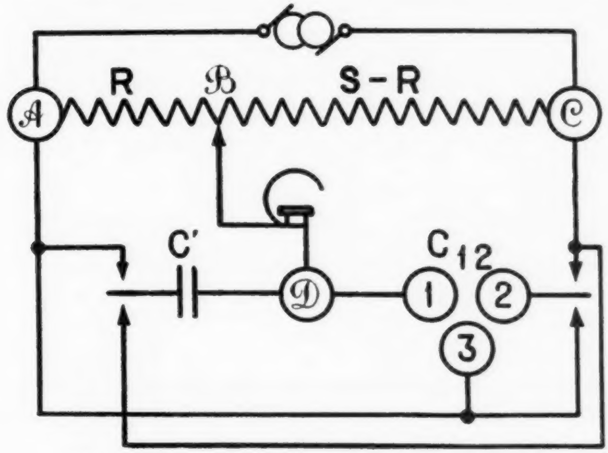


Fig. 3—Potentiometer Method for Direct Capacity

C', and the measurement is repeated with the required direct capacity and standard interchanged. Let R', R'' be the resistances required in arm $\mathcal{A}\mathcal{B}$ of the bridge for the first and second balance, then, S being the total slide wire resistance and G_1 the grounded capacity of terminal 1:[4]

$$C_{12} = \frac{R'}{R''} C',$$

$$G_1 = \frac{S - R''}{R''} C'.$$

This ratio method requires for the bridge a variable or slide wire resistance and a constant condenser, and it may be employed as an improvised bridge, when sufficient variable capacity is not available for the Colpitts method. Not being a substitution method, however,

[4] See appendix, section 3.

greater precautions are necessary for accurate results. There must be no initial direct capacity in arm $\mathcal{CD}$, or a correction will be required. Possibly variable capacity ratio arms would be preferable to resistances.

Null-Impedance Bridge Method for Direct Capacity, Fig. 4

Assuming that the electron tube supplies the means of obtaining an invariable true negative resistance, Fig. 4 shows a method which determines any individual direct capacity from a single bridge setting. The bridge arms are replaced by a Y network made up of two resist-

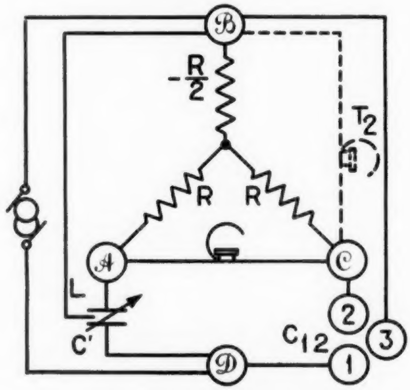


Fig. 4—Null-Impedance Bridge Method for Direct Capacity

ances R, R and a negative resistance $-R/2$; the Y has then a null-impedance between corner $\mathcal{B}$ and corners $\mathcal{A}$, $\mathcal{C}$ connected together[5]. The three terminals 1, 2, 3 of the network to be measured are connected to corners $\mathcal{D}$, $\mathcal{C}$, $\mathcal{B}$ and a balance obtained by adjusting the variable standard condenser C'. Then $C_{12} = C'$ regardless of the direct capacities associated with C_{12} and C', since these capacities either are short-circuited between corners $\mathcal{B}$, $\mathcal{A}$ or $\mathcal{B}$, $\mathcal{C}$ or are between corners $\mathcal{B}$, $\mathcal{D}$ and thus outside of the bridge.

Correct adjustment of the negative resistance may be checked by observing whether there is silence in telephone T_2 after the balance has been obtained. Assuming invariable negative resistance, this test need be made only when the bridge is set up, or there is a change in frequency. The bridge may be given any ratio Z_1/Z_2 by employing a Y made up of impedances Z_1, Z_2, and $-Z_1 Z_2/(Z_1 + Z_2)$.

[5] See appendix, section 4, which also describes a transformer substitute for the Y.

Maxwell Discharge Method[6], Fig. 5

Connect the terminals between which the direct capacity C_{12} is required, to A, B and the remaining accessible terminals of this electrical system to D. The adjustable standard capacity is C' and any associated direct capacities in this standard are shown as C'',

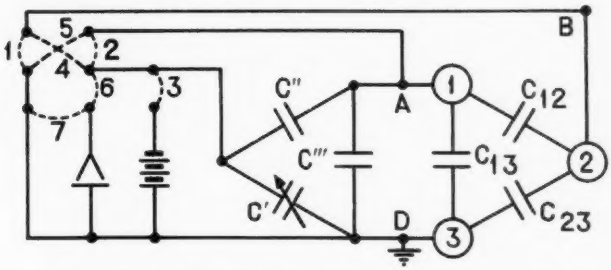


Fig. 5—Maxwell Discharge Method for Direct Capacity

C'''. If C_{12} is a direct capacity to ground, interchange C' and C_{12}. Balancing involves the following repeated cycle of operations:

1. Make connections 1, 2, 3 and 7 for an instant (thus charging C_{12}, C_{13}, C''', C' and discharging the electrometer).
2. Make connections 4, 5 and then 6 (to discharge condensers C_{13}, C''', mix charges of C_{12}, C' with polarities opposed and connect electrometer).
3. Adjust C' to reduce the electrometer deflection when the cycle is again repeated.

When a null deflection is obtained $C_{12} = C'$; the required direct capacity is equal to the standard direct capacity irrespective of the magnitudes of the four associated direct capacities. If all capacities are free of leakage and absorption, this remarkable method accurately compares two direct capacities by means of a single null setting, and it requires the irreducible minimum amount of apparatus.

Balanced-Terminal Capacity Measurement, Fig. 6

This is defined as the direct capacity between two given terminals with all other terminals left floating and ignored, after a hypothetical redistribution of the total direct capacity from the given pair of terminals to every third terminal which balances the two sides of the pair. The balanced-terminal capacity, as thus defined, is equal to the direct capacity between the pair augmented by one-quarter of

[6] Electricity and Magnetism, v. 1, p. 350 (ed. 1892).

the grounded capacity of the pair, neither of which is changed by the assumed method of balancing.

As illustrated in Fig. 6, terminals 1, 2 are the given pair and terminal 3 includes all others, assumed to be connected together. A bridge ratio of unity is employed, and the entire bridge is shielded from ground with the exception of corners $\mathcal{C}$, $\mathcal{D}$ which are initially balanced

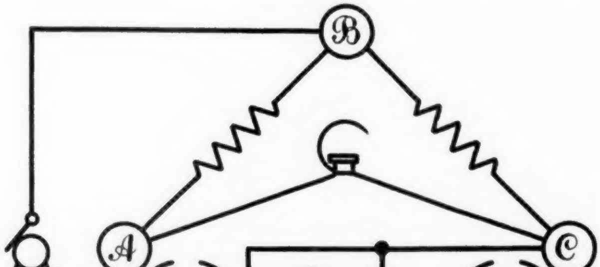


Fig. 6—Bridge for Determining Hypothetical Capacity Between Two Terminals with Other Terminals Balanced and Ignored

to ground within the range of variable condenser I. The following two successive balances are made:

1. With contacts a, a' closed and b open, balance is secured by varying condenser I (the total capacity of which is constant) giving the reading C' for its direct capacity in parallel with terminals 1, 3.
2. With contacts a, a' open and b closed, balance is obtained by varying condenser II, obtaining the reading C'' for its direct capacity in $\mathcal{A}\mathcal{D}$.

If C_o', C_o'' are the corresponding readings without the network, the balanced-terminal capacity C_b and the grounded capacity unbalance of the given pair of terminals are:[7]

$$C_b = 2\,(C'' - C_o''),$$
$$G_2 - G_1 = 2\,(C' - C_o').$$

[7] See appendix, section 5.

Any failure to adjust condenser I to perfectly balance the given pair of terminals will decrease the measured capacity C_b. This fact may be utilized to measure the capacity with the second bridge arrangement alone (contacts a, a' open and b closed) by adjusting condenser I so as to make the reading C'' of condenser II a maximum. This procedure presents no difficulty, since the correct setting for condenser I lies midway between its two possible settings for a balance with any given setting of condenser II; furthermore, C'' is not sensitive to small deviations from a true balance in C'.

Balanced-terminal capacity is of practical importance as a measure of the transmission efficiency to be expected from a metallic circuit, if it is subsequently transposed so as to balance it to every other conductor. In practice, when the unbalance of the section of open wire or cable pair, which is being measured, is relatively small, it is sufficient to set condenser I, once for all, to balance the bridge itself and ignore the unbalance of the pair. This favors an unbalanced pair, however, by the amount $(G_2 - G_1)^2/4\ (G_{12} + G_{CD})$ where $G_{12} + G_{CD}$ is the grounded capacity of the pair augmented by that of the bridge.[8] For rapid working, condenser II is graduated to read $2C''$ and by auxiliary adjustment C''_0 is made zero, so that the required capacity is read directly from the balance.

Additional Methods of Measuring Direct Capacity

Measurement of the capacity between the terminals, taken in pairs with all the remaining terminals left insulated or floating, gives $n(n - 1)/2$ independent results, from which all the direct capacities may be derived by calculation of certain determinants[9]. Practically, however, we are in general interested in determining individual direct capacities from the smallest possible number of measurements, and the first step is naturally to connect all of the remaining conductors together, so as to reduce the system to two direct capacities in addition to the one the value of which is required. Three measurements are then the maximum number required, and we know that two, or even one, is sufficient if particular devices are employed.

The three measurement method of determining direct capacities from the grounded capacities of the two terminals taken separately G_1, G_2, and together G_{12}, is given by Maxwell.[10] If $G_1 = C'$, $G_{12} = C' + C''$, and $G_2 = C'' + C'''$,

[8] See appendix, section 6.

[9] See appendix, section 7.

[10] *Ibid.*, p. 110.

then

$$\begin{aligned} C_{12} &= \tfrac{1}{2}\,(G_1 + G_2 - G_{12}) \\ &= \tfrac{1}{2}\,C''', \end{aligned}$$

which indicates a method by which large grounded capacities can be balanced against three variable capacities, only one of which need be calibrated, and that one need be no larger than the required direct capacity.

Two-setting methods, as illustrated by the Colpitts and potentiometer methods, rest upon the possibility of connecting one of the associated direct capacities between opposite corners of the bridge where it is without influence on the balance, and not altering any associated direct capacity introduced into the working arms of the bridge. Numerous variations of these methods have been considered which may present advantages under special circumstances. Thus, if conductors 1, 2, 3 of Fig. 7 are in commercial operation, and it is not permissible to directly connect two of them together, a double bridge might be employed with a testing frequency differing from

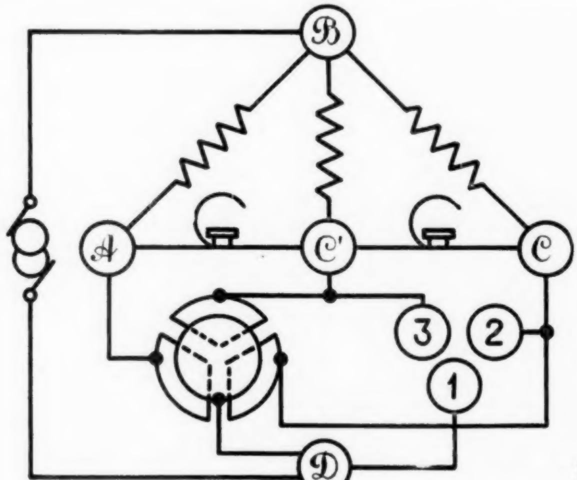


Fig. 7—Double Bridge for Direct Capacity

that of operation. A telephone is shown for each ear, and a constant total direct capacity is divided between the three branches in the proportion required to silence both telephones.

One-setting methods attained ideal simplicity in the Maxwell discharge method, but we found it necessary to use alternating current

methods, and here negative resistances make a one-setting method at least theoretically possible, as explained above. Of possible variations it will be sufficient to refer to the ammeter method Fig. 8. Termi-

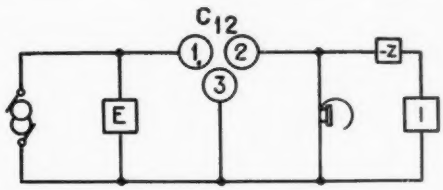


Fig. 8—Ammeter Circuit for Determining Direct Capacity

nals 1 and 2 of the required direct capacity C_{12} are connected to the voltmeter and ammeter terminals, respectively, and all other terminals go to the junction point at 3. Then

$$C_{12} = \frac{I}{2\pi f E},$$

provided the ammeter actually has negligible impedance. The method is well adapted for rapid commercial testing. The ammeter impedance may be reduced to zero by a variable negative impedance device $(-Z)$, adjusted to reduce the shunted telephone to silence.

Shielding

In the discussion of the bridge, it has been assumed that the several pieces of apparatus forming the six branches of the bridge have no mutual electrical or magnetic reaction upon each other, except as indicated. In general, however, a balance will be upset by changes in position of the pieces of apparatus, or even by movements of the observer himself, whereas these motions cannot affect any of the mutual reactions which have been explicitly considered. The skillful experimenter, understanding how these variations are produced by the extended electric and magnetic fields, will anticipate this trouble and take the necessary precautions, possibly without slowing down his rate of progress.

Where hundreds of thousands of measurements are to be made, however, substantial savings are effected by arranging the bridge so that reliable measurements can be made by unskilled observers, and here it is necessary to shield the bridge so that any possible movements of the observer and of the apparatus will not affect the results. Magnetic fields of transformers are minimized by using toroidal coils with iron cases. Electrostatic fields are shielded by copper cases;

the principles of shielding were explained in an earlier paper,[11] Fig. 13 of that paper showing the complete shielding of the balance as constructed for the measurement of direct capacity by the Colpitts method. Over five million capacity and conductance measurements have been made with the shielded capacity and conductance bridge and in a forthcoming paper Mr. G. A. Anderegg will give details of actual construction of apparatus and of methods of operation as well as some actual representative results.

Direct Admittance Measurements

For simplicity, the preceding definitions and methods of measurement have been described in terms of capacity, but everything may be generalized, with minor changes only, for the definition and measurement of direct admittances with their capacity and conductance components. The essential apparatus change is the addition, in parallel with the variable capacity standards employed, of a variable conductance standard, which shifts direct conductance from one side of the bridge to the other, without changing the total reactance and conductance in the two sides of the bridge. This may be practically realized in a great variety of ways as regards details, which it will suffice to illustrate by Fig. 9, where C', C'', C''', G', G''. indicate the

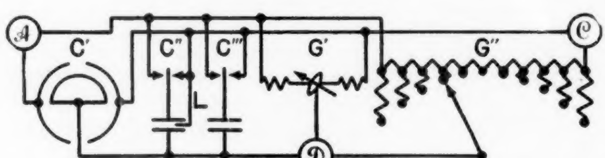


Fig. 9—Variable Direct Conductance and Capacity Standard for Direct Admittance Bridge

continuously variable capacity and conductance standards with enough step-by-step extensions to secure any desired range.

For the continuously variable conductance standard a slide wire is represented, with a slider made up of two hyperbolic arcs so proportioned that, as the slider is moved uniformly in a given oblique direction, conductance is added uniformly on the left and just enough of the wire is short-circuited to produce an equal conductance decrease on the other side. The arcs are portions of the hyperbola $xy = (L^2 - S^2)/4$, where L, S are the total length of the wire and of the portion to be traversed by the slider, and the coordinate axes are

[11] The Shielded Balance, El. W., 43, 1904 (647–649).

the slide wire and the direction of the motion of the slider as oblique asymptotic axes.[12] $L = GS/g = 4G/\rho(G^2 - g^2)$, where G is the total conductance and $(G \pm g)/2$ the limiting direct conductance on either side.

If an ordinary slider replaces the hyperbolic arc slider, and the scale reading is made non-uniform so as to give one-half of the difference between the direct conductances $\mathcal{A}$ to $\mathcal{D}$ and $\mathcal{C}$ to $\mathcal{D}$, the conductance standard will still give absolutely correct results with the Colpitts method, provided the bridge ratio is unity. This simplification in connection with the balancing capacity I of Fig. 6 would, however, not be strictly allowable. For improvised testing we have found it sufficient to use two equal resistances (R) with a dial resistance (r) in series with one of them, and take the defect of conductance introduced by the dial resistance as equal to r/R^2 or to $10^{-2}r$, $10^{-1}r$, r, micromho according as R was made 10000, 3162, or 1000 ohms.[13]

For a step-by-step conductance standard, Fig. 9 shows a set of 10 equal resistances, connected in series between corners $\mathcal{A}$, $\mathcal{C}$, to the junction points of which there is connected a parabolic fringe of resistances, the largest of which is 2.5 times each of the ten resistances. With this arrangement the direct conductance in $\mathcal{A}\mathcal{D}$ may be adjusted by ten equal steps, beginning with zero, while the conductance in $\mathcal{C}\mathcal{D}$ is decreased by equal amounts to zero. The total resistance required for this conductance standard is only 21/25 of the resistance required to make a single isolated conductance equal to one of the ten conductance steps; the ratio may be reduced to 1/2 by doubling the number of contacts,[14] and using one fringe resistance for all positions. Resistance may be still further economized by using as high a total conductance as is permissible in the bridge, and securing the required shift in conductance from a small central portion of the parabolic fringe.

Fig. 9 shows the variable capacity standards as well as the variable conductance standards and a few practical points connected with the capacity standards may be mentioned here.

The revolving air condenser standard has two fixed plates connected to $\mathcal{A}$ and $\mathcal{C}$, so that the capacity will increase as rapidly on one side as it decreases on the other side. Since perfect constancy of the total capacity is not to be expected, on account of lack of perfect mechanical uniformity, the revolving condenser should be calibrated to read

[12] See appendix, section 8.
[13] See appendix, section 9.
[14] See appendix, section 10.

one-half of the difference between the capacities on the two sides, as explained above in connection with conductance. The capacity sections employed to extend the range of the revolving condenser include both air condensers C' and mica condensers C'', the latter being calibrated by means of the air condensers and the conductance standard.

A novel feature of our standard air condensers is a third terminal called the leakage terminal, and indicated at L in Figs. 4, 9. Attached to it are plates so arranged that all leakages either over, or through, the dielectric supports from either of the two main terminals, must pass to the leakage terminal. There can be no leakage directly from one of the main terminals to the other. There is thus no phase angle defect in the standard direct capacity due to leakage, and that due to dielectric hysteresis in the insulating material is reduced to a negligible amount by extending the leakage plates beyond the dielectric, so as to intercept practically all lines of induction passing through any support. This leakage terminal is connected to corner C of the bridge; in the revolving condensers, it is one of the fixed plates.

Direct Impedance Measurements

The reciprocal of a direct admittance is naturally termed a direct impedance; substituting impedance for capacity, the definition of direct capacity, given above, becomes the definition of direct impedance. The complete set of direct impedances constitutes an exact, symmetrical, physical substitute for any given electrical system. Direct impedances are often, in whole or in part, the most convenient constants since many electrical networks are made up of, or approximate to, directly connected resistances and inductances. To make direct impedance measurements which will not involve the calculation of reciprocals, we naturally employ inductance and resistance standards in series, the associated direct impedances being eliminated as with direct capacities.

Conclusion

It has been necessary to preface the description of methods of measuring direct capacities by definitions and a brief discussion, since direct capacities receive but scant attention in text-books and handbooks. By presenting direct capacities, direct admittances, and direct impedances as alternative methods of stating the constants of the same direct network, employed as an equivalent substitute for any given electrical system, it is believed the discussion and measure-

ment of networks has been simplified. In another paper the terminology for admittances and impedances will be still further considered, together with their analytical correlation.

APPENDIX

In explaining the different methods of measuring direct capacities it is necessary to start with a clear idea of what direct capacities are, and to make use of the additive property, but it is not necessary to go into any comprehensive discussion of direct capacities. Accordingly, the mathematical treatment of direct capacities has been reserved for another paper, but it seems desirable to append to the present paper proofs of the analytical results given in this paper, since the method of approach giving the simplest proof is not always perfectly obvious.

(1) Reducing the number of terminals which are considered accessible, by ignoring terminals $p, q, r, \ldots$, changes the direct and grounded capacities from (C_{ij}, G_i) to (C'_{ij}, G'_i), the latter being expressed in terms of the former as follows:

$$C'_{ij} = \frac{-\begin{vmatrix} -C_{ij} & -C_{ip} & -C_{iq} & \cdot\cdot \\ -C_{jp} & G_p & -C_{pq} & \cdot\cdot \\ -C_{jq} & -C_{pq} & G_q & \cdot\cdot \\ \cdot & \cdot & \cdot & \cdot\cdot \end{vmatrix}}{\begin{vmatrix} G_p & -C_{pq} & \cdot\cdot \\ -C_{pq} & G_q & \cdot\cdot \\ \cdot & \cdot & \cdot\cdot \end{vmatrix}}$$

$$G'_i = -C'_{ii}$$

where C'_{ii} is given by formula above and $G_i = -C_{ii}$.

To check these formulas note that on substituting $(G_i, -C_{ij})$ for Maxwell's (q_{ii}, q_{ij}) in his equations (18)[15] the coefficients form an array in which the grounded capacity G_i is the ith element in the main diagonal and $-C_{ij}$ is the element at the intersection of row i, column j. The array may be supposed to include every terminal symmetrically by considering the earth's potential as being unknown and writing down the redundant equation for the charge on the earth. Let the charge be zero on terminal j and on all concealed terminals; let there be a charge on terminal i and an equal and opposite charge on all the remaining accessible terminals, connected together to form a single terminal k. Now taking the potential of j as the zero of reference

[15] *Ibid.*, p. 108.

and calculating the potentials of i and k and then allowing the direct capacity between j and k to become infinite, the direct capacity between terminals i and j is $C_{ij} = - Lim\ (C_{jk}\ V_k / V_i)$. This gives the above formula for C'_{ij}, with $- C'_{ii}$ as a special case. This method is an electrostatic counterpart of the ammeter method shown in Fig. 8 on page 29.

If there is but one ignored terminal the determinant solution takes on a simple form from which Rules 1 and 2 and Fig. 1 may be checked.

If all but two terminals are ignored the equivalent direct network is reduced to a single direct capacity. When, for each pair of terminals, this capacity C'_{ij} is known, from measurements or from calculations, the direct capacities between the terminals may be derived by means of the following formulas

$$C_{ij} = 2\,\frac{D_{ij}}{D}$$

$$G_i = - C_{ii} = - 2\,\frac{D_{ii}}{D}$$

where D_{ij} is the cofactor of the element in row i column j of the determinant

$$D = \begin{vmatrix} 0 & S_{12} & S_{13} & .. & 1 \\ S_{12} & 0 & S_{23} & .. & 1 \\ S_{13} & S_{23} & 0 & .. & 1 \\ . & . & . & .. & \\ 1 & 1 & 1 & .. & 0 \end{vmatrix}$$

which has zeros in the main diagonal, a border of ones in the last row and column, while the other elements are $S_{ij} = 1/C'_{ij}$, that is, the reciprocals of the given capacities. The S's form a complete symmetrical system of network constants; Maxwell's coefficients of potential p_{ii} with the two suffixes the same are the same quantities, but he employs only those coefficients of this type which are associated with the earth, his system being completed by adding the coefficients with different suffixes. By starting with Maxwell's results the above formula may be deduced, but more direct proofs, both physical and mathematical, will be given in the theoretical paper referred to at the end of the present paper.

The purpose of this section of the appendix is achieved if the determinant solutions are made so clear as to be available for use in any particular case.

(2) Starting with the bridge alone balanced at reading C° the other two settings involve, in the capacity standard, increases in the direct

capacity on the left of $(C' - C^\circ)$ and $(C'' - C^\circ)$, with equal decreases on the right. Therefore

$$C_{12} + C_{14} + (C' - C^\circ) = -(C' - C^\circ) + C_{13}$$
$$C_{14} + (C'' - C^\circ) = -(C'' - C^\circ) + C_{13} + C_{12}$$

and adding gives the value of $(C_{13} - C_{14})$.

(3) The condition of equal impedance ratios on the two sides, as required for a balance, gives, for both the switches up and down,

$$R'(G_1 - C_{12} + C') = (S - R')\,C_{12},$$
$$R''\,G_1 = (S - R'')\,C',$$

respectively, from which the expressions for C_{12} and G_1 follow.

(4) The Y of Fig. 4 has unusual properties because the total conductance connecting the concealed branch-point of the Y to the three bridge corners $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ is zero. Thus the conductance between any one corner and the remaining two corners joined together is infinite, or in other words, the Y acts as a short circuit under all these three conditions. On the other hand, if corner $\mathcal{A}$, $\mathcal{B}$, or $\mathcal{C}$ is left floating and ignored the conductance between the other two corners is $2/R$, $1/2R$ or $2/R$, respectively, and the Y is not a short circuit. These statements are verified at once by applying the familiar expressions for resistances in parallel and in series.

On account of the unusual behavior of the Y, even when taken alone, it is not immediately apparent how it will affect the operation of the bridge of Fig. 4 with direct capacities between corners $\mathcal{A}\mathcal{B}$ and $\mathcal{B}\mathcal{C}$. For this reason it is highly desirable to find an equivalent network the behavior of which is more readily comprehended. It is not feasible to employ the delta network which is equivalent to the Y for this has indeterminate characteristics, being made up of three infinite conductances, only two of which have the same sign. We may, however, make use of the Y which is equivalent to the original Y and direct capacities C_{AB} and C_{BC} taken together. This is found as follows: Any admittance delta may be replaced by a star having admittances equal to the sum of the products of the delta admittances taken in pairs divided by the opposite delta admittance. Applied to the delta of Fig. 1, we find that the star that is equivalent has the capacities

$$C''_{14} = \frac{S}{C_{24}\,C_{34} + G_4\,C_{23}}$$
$$C''_{24} = \frac{S}{C_{34}\,C_{14} + G_4\,C_{13}}$$
$$C''_{34} = \frac{S}{C_{14}\,C_{24} + G_4\,C_{12}}$$

where

$$S = C_{14}C_{24}C_{34} + C_{14}C_{24}(C_{13}+C_{23}) + C_{24}C_{34}(C_{12}+C_{13}) + C_{34}C_{14}(C_{12}+C_{23}) \\ + G_4(C_{12}C_{23} + C_{23}C_{13} + C_{12}C_{13}),$$

which, upon substituting the value of G_4, is the sum of 16 terms, each of which is the product of three capacities, every combination of three capacities being included except the four cases in which the three capacities would form a closed circuit. By allowing the capacities to be complex quantities, any admittances are covered by the formulas.

If $G_4 = 0$

$$\frac{C''_{14}}{C_{14}} = \frac{C''_{24}}{C_{24}} = \frac{C''_{34}}{C_{34}} = \frac{S}{C_{14}C_{24}C_{34}}$$

or the new Y arms present the same ratios as the original Y arms taken alone; that is, the direct capacities C_{AB}, C_{BC} of Fig. 4 have no effect on the bridge ratio. Thus the constancy of the bridge ratio holds for all null-impedance bridges regardless of the ratio Z_1/Z_2 and of the nature of the direct admittances from corners A and C to B.

If $G_4 = 0$ and also $C_{24} = C_{14}$ and $C_{12} = 0$, then

$$C''_{14} = C''_{24} = -\tfrac{1}{2}C''_{34} = C_{14} + \tfrac{1}{2}(C_{13} + C_{23}).$$

Applying this to Fig. 4, which is possible since the bridge ratio is unity, we find that the three arms of the equivalent Y may be considered as being made up of resistances and capacities in parallel. The resistances are R, R, $-R/2$ and the associated capacities C, C, $-2C$, where R is the original resistance in the Y and C is one-half the sum of the two actual direct capacities from $\mathscr{B}$ to $\mathscr{A}$ and from $\mathscr{B}$ to $\mathscr{C}$. The equivalent bridge thus obtained has ratio arms made up of ordinary resistances and capacities and therefore Fig. 4 used as a bridge can present no unexpected characteristics; the negative resistances and capacities of the equivalent Y merely affect the current supplied to the bridge.

An ideal transformer, if such a device existed, might replace the Y, for it would maintain a constant ratio between the currents in the two windings and act as a short circuit when the bridge is balanced. To determine the error when an actual transformer with impedances Z_p, Z_s, Z_{ps} is employed, take the general expression for the ratio of the capacities derived above which is

$$\frac{C''_{14}}{C''_{24}} = \frac{C_{34}C_{14} + G_4C_{13}}{C_{24}C_{34} + G_4C_{23}}.$$

Change to admittances by substituting Y for C and G throughout. Assume the transformer replaced by its equivalent conductance star so that

$$Y_{14} = \frac{1}{Z_p + Z_{ps}},$$

$$Y_{24} = \frac{1}{Z_s + Z_{ps}},$$

$$Y_{34} = -\frac{1}{Z_{ps}}, \text{ and by addition}$$

$$Y_4 = \frac{Z_{ps}^2 - Z_p Z_s}{(Z_p + Z_{ps})(Z_s + Z_{ps}) Z_{ps}}.$$

Substituting these values the expression for the actual ratio of the bridge arms becomes

$$\frac{Y_{14}''}{Y_{24}''} = \frac{Z_s + Z_{ps} + (Z_p Z_s - Z_{ps}^2) Y_{13}}{Z_p + Z_{ps} + (Z_p Z_s - Z_{ps}^2) Y_{23}}.$$

(5) When the bridge alone is balanced at readings C_o' and C_o'', let C_{CD} and G_{CD} be the direct capacity between corners $\mathcal{C}$ and $\mathcal{D}$ and the total direct capacity between these corners and ground. Since G_{CD} is balanced, the effective direct capacity between corners $\mathcal{C}$, $\mathcal{D}$ when earth is ignored, is by Fig. 1, $(C_{CD} + G_{CD}/4)$. Now connect the three terminals 1, 2, 3, as shown with direct capacities C_{12}, $G_1 - C_{12}$, $G_2 - C_{12}$; G_1, G_2 being the grounded capacities of terminals 1 and 2. The first balance with the reading C' requires the equality of the total capacity added on each side, *i.e.*,

$$G_1 - C_{12} + (C' - C_o') = G_2 - C_{12} - (C' - C_o')$$

or

$$G_2 - G_1 = 2\,(C' - C_o')$$

For the second balance ground may again be considered an ignored terminal, and since terminals 1 and 2 have been balanced to ground, and their total direct capacity to ground is $G_{12} = G_1 + G_2 - 2C_{12}$, the effective direct capacity added to the bridge between corners $\mathcal{C}$ and $\mathcal{D}$ is $C_b = C_{12} + G_{12}/4$. Equating the added capacities on the two sides of the bridge when balanced at the reading C'', we obtain $C_b = 2\,(C'' - C_o')$.

The direct capacity between $\mathcal{C}$ and $\mathcal{D}$, when ground is considered an accessible terminal, is assumed to be absolutely independent of the setting of the condenser I. To actually meet this condition will require some attention in the design of the variable condenser.

(6) Here the bridge itself is supposed to have equal direct capacities from corners $\mathcal{C}$ and $\mathcal{D}$ to ground, while the added terminals 1 and 2 have different direct capacities to ground, the difference being $(G_1 - G_2)$, while the total direct capacity to ground is $(G_{12} + G_{CD})$. Now two capacities in series may be replaced by their product divided

by their sum, which is equal to one-fourth of the sum minus the square of the difference divided by four times the sum. The correction due to the difference is thus $(G_2 - G_1)^2/4(G_{12} + G_{CD})$, as stated.

(7) These determinants are given at the end of the first paragraph of this appendix. These expressions for the direct capacity are of more special interest in the analytical discussion of networks.

(8) Assume that a wire resistance is to be employed and that a sliding contact is to intercept such an amount of resistance that the equivalent conductance will vary directly with the motion of the slider carrying the contact point. Then if the wire is straight and the intercepted portion is of length x and the slider motion is rectilinear and its extent is y the relation which holds between them is xy = constant, the value of the constant depending upon the units employed.

In the paper it is assumed that the total conductance G, the total shifted conductance g, and the resistance of unit length of the slide wire ρ are given; the total length of wire L and the portion traversed by the slider S are then calculated. The arc employed, for each half of the slider of Fig. 9, extends equally both ways from the vertex to the points where the values of x and y are $(L \pm S)/2$, on the hyperbola $xy = (L^2 - S^2)/4$. Substituting for L and S the values given in the paper, it will be found that this range of x actually gives the range of conductance $(G \pm g)/2$, as required.

(9) The exact defect in conductance is

$$\frac{1}{R} - \frac{1}{R+r} = \frac{r}{R\,(R+r)} = \frac{r}{R^2}\left(1 - \frac{r}{R} + \ldots\right)$$

(10) At mid-point the total conductance due to the five resistances (R) on each side, taken in parallel, is $2/5R$ and to give this same conductance an end fringe must have the resistance $2.5R$. Assume a parabolic fringe having the resistance $(5 - n)^2\, R/10$ at the point connected to $\mathcal{A}$ and $\mathcal{C}$ by resistances nR and $(10 - n)R$. This gives a Y network and by Fig. 1 the equivalent direct conductances are $(10 - n)/25R$, $n/25R$, $(5 - n)^2/250R$ between $\mathcal{DA}$, $\mathcal{DC}$, $\mathcal{AC}$ respectively. The sum of the first two is constant and the first decreases by equal steps, of $1/25R$ each, to zero as the second increases. The parabolic fringe, therefore, gives the required conductances.

The total resistance in the chain of ten resistances is $10R$, in the fringe $11R$, and in the largest single fringe $2.5R$. With the complete fringe the total required is $(10 + 11)\, R = 21R$; with a single fringe, subdivided as required, only $(10 + 2.5)\, R = 12.5R$ is required. Compared with $25R$, which would be required for one of the conductance steps, these resistances are $21/25$ and $1/2$.

The Relation of the Petersen System of Grounding Power Networks to Inductive Effects in Neighboring Communication Circuits

By H. M. TRUEBLOOD

THE purpose of this paper is to present a simple theoretical treatment of those features of the Petersen method of grounding a power network which are of principal interest from the standpoint of inductive effects in neighboring communication circuits. In this method, the neutral of the system is grounded through an inductance which is in resonance, at the fundamental frequency, with the total direct capacity of the system to ground. The theory of the behavior of a power system thus grounded at times of accidental faults to earth has been developed by Petersen in a paper published in 1919,[1] in which the results of field tests and of operating experience with an installation in Germany are also described. The method has also found application in other places in Europe, chiefly in Italy and Switzerland. It does not appear in any of these cases that inductive interference was a factor requiring, or at any rate receiving, consideration. In fact, it does not seem that either Petersen himself, or other engineers in Europe who have made use of his scheme, have considered it except as a method of protecting power systems from the effects of accidental grounds.

The features of the method that are of interest from the viewpoint of inductive interference relate both to normal operating conditions of the power system and to the phenomena which occur when a phase of the system is grounded. With regard to the former, it is principally, though not entirely, the effect of the neutral reactor on the harmonics of frequencies within the voice range that require examination; with respect to the latter, the things of chief, though not exclusive, importance, are the ground currents and unbalanced voltages to ground of fundamental frequency, which are possible sources of disturbance in exposed communication circuits.

These features, particularly those concerned with effects at fundamental frequency, are more or less closely related to questions of primary importance from the standpoint of power system operation. It is impossible that this should not be the case. Accidental disturbances of a character which may interrupt service or endanger apparatus or equipment in a power system may produce inductive

[1] W. Petersen, Elektrotechnische Zeitschrit, 40, pp. 5-7 and 17-19, 1919; Sci. Abs. B, Nov. 29, 1919.

disturbances in neighboring communication circuits, and the question of grounding the neutral, in whatever manner, or of leaving it isolated, exists because of its bearing on the avoidance or limitation of such power system disturbances. Thus a method of grounding the neutral, or any other method of power system operation designed to limit the extent or the severity of accidental disturbances, must necessarily possess importance with respect to inductive effects in exposed communication circuits.

It does not seem necessary, therefore, to apologize for the discussion, in the first section of the paper, of the behavior of the power system at times of faults to earth. In this section, an explanation is given of the principal characteristic effect of the reactor which differs in some respects from that set forth by Petersen in the paper already referred to. The matter of transient over-voltage on a non-grounded phase is also examined in this section, and the bearing of these and the earlier considerations on inductive effects is discussed.

In the second section of the paper, the behavior of the power system with reactor under normal operating conditions is discussed with reference to noise and other inductive effects in neighboring communication circuits.

1. Effects with a Grounded Phase on the Power System

1. *Action of Coil in Suppressing Arcs to Ground*

Referring to Fig. 1, in which, and in the following discussion it is assumed that the three admittances to ground are equal, the admittance current through the fault from the two sound phases is

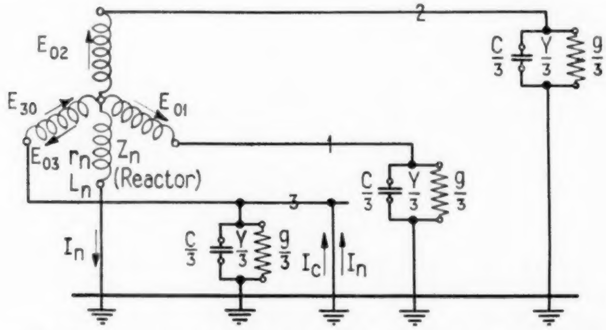


Fig. 1—Single-Phase System with Neutral Grounded Through Reactor. Admittances to Ground Assumed Balanced

$$I_c = \frac{Y}{3}(E_{01} - E_{03}) + \frac{Y}{3}(E_{02} - E_{03}) = \frac{Y}{3}(E_{01} + E_{02} - 2E_{02}),$$

$$= Y\,E_{30} = (g + jC\omega)\,E_{30}. \tag{1}$$

The current through the coil and fault is

$$I_n = \frac{E_{30}}{Z_n} = \frac{E_{30}}{r_n^2 + \omega^2 L_n^2}(r_n - j\omega L_n),$$

or, neglecting $\frac{r_n^2}{\omega^2 L_n^2}$ in comparison with unity,

$$I_n = \frac{E_{30}}{\omega^2 L_n^2}(r_n - j\omega L_n).$$

Thus the total fault current is

$$I_c + I_n = I_f = E_{30}\left[g + \frac{r_n}{\omega^2 L_n^2} + j\left(\omega C - \frac{1}{\omega L_n}\right)\right],$$

and, if the coil is adjusted for resonance,

$$I_f = E_{30}\left(g + \frac{r_n}{\omega^2 L_n^2}\right),$$

$$= \frac{E_{30}}{\omega L_n}\left(\frac{r_n}{\omega L_n} + \frac{g}{\omega C}\right). \tag{2}$$

On comparison of this expression with the above equation (1) for the charging current, which constitutes the fault current if the system is isolated, it is seen that, if the losses in the system are small, the effect of the coil is to reduce the magnitude of the current in the fault approximately in the ratio of $\left(\frac{r_n}{\omega^2 L_n^2} + g\right)$ to $|Y|$, *i.e.*, neglecting terms of the second and higher orders, in the ratio $\left(\frac{r_n}{\omega L_n} + \frac{g}{\omega C}\right)$ to unity. Further, as equation (2) shows, the phase of the fault current coincides with that of the voltage impressed between the faulty wire and ground to the degree of approximation here used, *i.e.*, to the second order of small quantities. (The bracket on the right-hand side of (2), if written in full, would include quadrature terms in the square and higher even powers of $r_n/\omega L_n$.) With the system isolated, the phase displacement is nearly 90°.

The action of the coil may be described as a transfer of the charging current from the fault to the coil, leaving nothing but the component of current to supply losses at the fault. With suitable design of the coil, this energy current can be made small.

The coincidence in phase of the fault current and the voltage impressed by the transformer on the faulty wire, together with the small magnitude of the former, are very favorable to the suppression of the

arc. Following its extinction, there is a further action of the resonant system consisting of the coil and the total capacity to ground which acts in such a way as to prevent any over-voltage and to restore the normal potentials to ground *gradually*, thus tending to prevent the arc from restriking. This action is as follows:

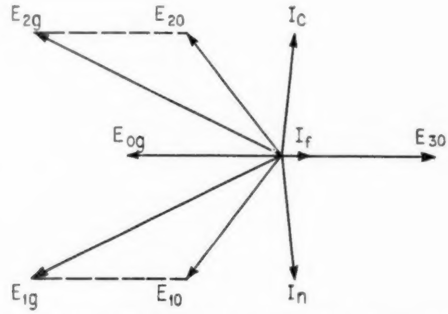


Fig. 2—Vector Diagram Showing Relations of Voltages and Currents at and Following Extinction of Arc

Referring to Fig. 2, the vectors E_{10}, E_{20}, E_{30} represent the emfs. impressed between lines and neutral by the transformer bank. I_c, I_n, I_f are respectively the admittance current to the fault, the current supplied by the reactor to the fault, and the total fault current. The fault is assumed to be on phase 3. The arc will go out when I_f passes through zero. At this instant, E_{30} is also zero and I_n and I_c have nearly their maximum values. Their instantaneous values are, however, exactly equal and opposite in the sense indicated by the arrows in Fig. 1, *i.e.*, regarded as currents fed to the fault by the two parallel circuits (1) coil-fault-faulty wire-E_{30} and (2) admittance of sound phases-fault-faulty wire-transformer bank. These instantaneous currents are exactly equal in magnitude and are in the same direction in the single series circuit consisting of coil, transformers, admittance to ground of the three phases in parallel, and ground. Thus the condition in this series resonant circuit at the instant of extinction is that of an established free oscillation, the energy of the oscillation being at this instant wholly electromagnetic.

The voltage across the reactor due to the current I_n, in the direction of E_{30} (Fig. 1) is represented in Fig. 2 by the vector E_{0_g}. This is 180° out of phase with E_{30} and initially of the same amplitude. At the instant the arc goes out, both are practically zero. As the oscillation progresses, E_{0_g} dies away, due to damping, and the resultant

voltage of the faulty wire to ground, *viz.*, $E_{3g} = E_{30} + E_{0g}$, passes gradually back to the normal value E_{30}. At the same time the voltages to ground of the two sound phases (E_{1g}, E_{2g},) return to their normal values E_{10}, E_{20}. The ends of the three vectors, E_{3g}, E_{1g}, E_{2g}, may be thought of as sliding at equal rates along the line E_{30} and the dotted lines parallel to it.

The effectiveness of the action just sketched (it has been assumed that the frequency of the series resonant circuit is accurately that of the system fundamental), of course, depends on the accuracy of the tuning and the amount of damping. If the free period of the resonant circuit differs considerably from the fundamental period of the system, the impressing on the faulty wire of a voltage to ground in excess of normal may result, especially if the damping is small. The effect of inexact tuning is discussed by Petersen in the article referred to above. He describes some experiments in which the capacity of the power system to ground was varied some 15 or 20%, each way from the value corresponding to resonance, without apparent effect on the quenching action of the reactor.

2. *Transient Overvoltage on Sound Phase at Time of Grounding*

To simplify the following theoretical discussion a single phase system is treated. This is represented in Fig. 3. Referring to this

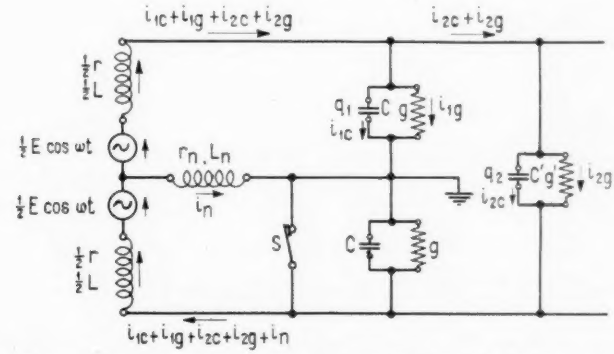


Fig. 3—Three-phase System with Neutral Grounded Through Reactor, with Fault to Ground on One Phase

figure, when one phase is grounded (represented by the closing of the switch S), the following equations, in which D denotes differentiation with respect to time, must be satisfied:

$$\Phi\,(D)\,i_n = \frac{E}{2}\cos\omega t$$

$$\frac{1}{g}\Phi\,(D)\,i_{1g} = \left[\frac{r}{2} + 2r_n + \left(\frac{L}{2} + 2L_n\right)D\right]\frac{E}{2}\cos\omega t$$

where

$$\Phi(D) = LC'_0\left(\frac{L}{2}+2L_n\right)D^3 + \left[C'_0(rL+2L_nr+2r_nL)+g'_0L\left(\frac{L}{2}+2L_n\right)\right]D^2$$
$$+\left[g'_0\,(rL + 2L_nr + 2r_nL) + C'_0\left(\frac{r^2}{2} + 2rr_n\right) + L + 2L_n\right]D$$
$$+\,g'_0\left(\frac{r^2}{2} + 2rr_n\right) + r + 2r_n$$
$$C'_0 = C + C',\ g'_0 = g + g'.$$

To solve the equations, the cubic equation $\Phi\,(D) = 0$ must be solved. An algebraic solution would be so cumbersome as to be impracticable. The following numerical values of the constants have therefore been inserted, as what is desired is a numerical solution representing the effect in a practical case:

$g = 0.37 \times 10^{-6}$ mho	$g' = 0.18 \times 10^{-6}$ mho
$C = 0.55 \times 10^{-6}$ farad	$C' = 0.30 \times 10^{-6}$ farad
$L_n = 6.4$ henries	$r_n = 200$ ohms
$L = 0.022$ henry	$r = 2.0$ ohms
$E = \sqrt{2} \times 26{,}400 = 37{,}350$ volts	$\omega = 377$

With these assumptions

$$\Phi\,(D) = 2.4 \times 10^{-7}D^3 + 29.4 \times 10^{-6}D^2 + 12.8\,D + 402,$$

of which the roots are

$$-\,31.4,\ -\,45.5 + j7{,}300,\ -\,45.5 - j7{,}300$$

which may be denoted by $-\,a'$, $-\,a + jb$, $-\,a - jb$ respectively. The resulting equations for i_{1g} and i_n are

$$i_{1g} = Pe^{-a't} + Qe^{-at}\sin\,(bt + \Theta) + gE\cos\omega t,$$

$$i_n = P'e^{-a't} + Q'e^{-at}\sin\,(bt + \Theta') + \frac{E}{4820}\sin\,(\omega t + 4°.7).$$

The relations between the two sets of arbitrary constants may be obtained by inserting these solutions in the following differential equation connecting i_{1g} and i_n:

$$i_{1g}/g - \left[r/2 + 2r_n + (L/2 + 2L_n)\,D\right]i_n = 0,$$

and the three independent arbitrary constants so found are determined by the following conditions when $t = 0$ (it is assumed that breakdown

occurs when the impressed voltage to ground, $E \cos \omega t/2$, is a maximum):

$$i_n = 0,$$

$$i_{1g} + i_{2g} + i_{1c} + i_{2c} = (g'_0 + C_0'D)\,\frac{i_{1g}}{g} = 0.0173 \text{ ampere},$$

$$q_1 + q_2 = \frac{i_{1g}}{g}\,C'_0 = 0.0216 \text{ coulomb}.$$

The numerical quantities in the second and third of these equations are respectively the total current supplied to the sound wire and the total charge on this wire at the instant of breakdown. They are obtained by solving the network of Fig. 3, with switch S open and taking instantaneous values when the impressed e. m. f. is a maximum. The resulting expressing for i_{1g} is

$$i_{1g} = 0.3 \times 10^{-6} e^{-31.4t} - 4.38 \times 10^{-3} e^{-45.5t} \cos 7300t$$
$$+ 0.37 \times 10^{-6} \times 37{,}350 \cos 377t.$$

The non-oscillatory term $0.3 \times 10^{-6} e^{-31.4t}$ is seen to be negligible compared to the others.

The voltage between the sound wire and ground is obtained by dividing the above result by $g = 0.37 \times 10^{-6}$, and is

$$v = 0.8e^{-31.4t} - 11{,}800e^{-45.5t} \cos 7{,}300t + 37{,}350 \cos 377t.$$

This equation is plotted for about 1½ cycles of fundamental frequency in Fig. 4. The non-oscillatory term is negligible. As will be seen, the maximum overvoltage is about 30%.

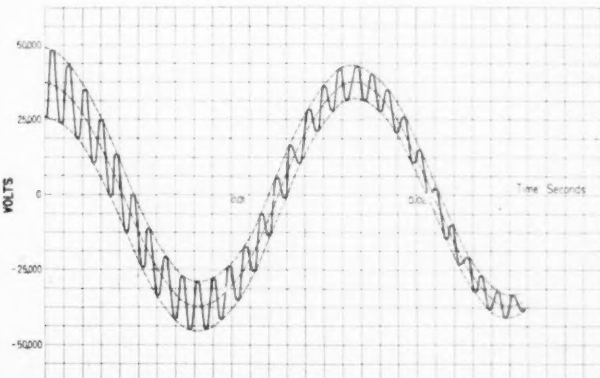


Fig. 4—Voltage from Sound Phase to Ground Following Extinction of Arc.

For comparison purposes, the voltage to ground of the sound phase with the reactor omitted (*i.e.*, with the system isolated) has been calculated, using the same constants as before. The result is

$$v' = -11,900e^{-45.8t} \cos 7310t + 37,350 \cos 377t.$$

This is practically identical with the earlier result; that is, the voltage to ground of the sound phase is practically the same with the reactor as with the system isolated.

3. Effects with Respect to Induction in Neighboring Communication Circuits

An estimate of the value of the Petersen coil must involve a comparison with other methods of grounding the neutral (including grounding through an infinite impedance, *i.e.*, the isolated system) or of otherwise limiting the effects of abnormal occurrences in a power system. As regards the induction of fundamental frequency voltages in exposed communication circuits, the methods of chief importance in such a comparison, at least so far as American practice is concerned, are that in which the neutral is grounded either directly or through a low resistance, and that in which the neutral is isolated.

When accidental grounds occur on a power system with neutral grounded through zero or a low impedance, the resulting heavy short circuit currents to ground may produce severe disturbances in exposed communication circuits. Owing to the fact that these disturbances are produced by electromagnetic induction in a circuit consisting of the communication conductor as one side and the earth as the other, they cannot be avoided by enclosing the communication conductors in lead-sheathed cable, even when this is placed underground.

With the Petersen coil, according to the explanation in the first part of this section, the neutral current of fundamental frequency due to a fault to ground is made equal to the charging current of the system to ground with one phase grounded, and this is generally a small fraction[2]—a few per cent. or less—of the neutral current in an identical system with neutral directly grounded.

The Petersen coil will thus in many cases largely prevent the electromagnetic inductive effects at fundamental frequency which appear when a fault to ground occurs on a system grounded solidly or through

[2] Exceptions to this statement exist in the case of extensive high voltage networks where, with a ground on one phase, the charging current to ground with isolated neutral may be of the same order of magnitude as the short circuit current with dead-grounded neutral if the fault is remote from a point of main power supply.

a low resistance. There will appear, however, electrically[3] induced voltages of fundamental frequency substantially identical with those that would occur with neutral isolated. Where the communication circuits are in underground cable, these voltages are of inappreciable magnitude, and with aerial cables (with metallic sheaths) their effects can in general be controlled without great difficulty. With open wire communication circuits, electrically induced voltages are of much more consequence. They may in some cases equal or exceed the voltages which would be induced electromagnetically with dead-grounded neutral. However, except perhaps in cases of long exposure to high voltage power circuits at close separations, their effects are generally much less severe than the electromagnetic effects, because of the smaller amount of energy transferred to the disturbed circuit. This is in general accordance with experience with open wire circuits exposed to power circuits of moderate voltage. As is explained in the next paragraph, the use of a Petersen reactor to ground the neutral may be expected to lessen the severity of inductive effects which would be experienced from an isolated system, by preventing additional parts of the power system from becoming involved.

As compared to the isolated system, the use of the Petersen coil in the connection from neutral to ground may be expected to have the advantage, according to the theory of the first subdivision of this section, of preventing the formation of an "arcing ground." As experience has shown, an arcing ground in an isolated system is frequently the cause of serious disturbances which may involve portions of a network remote from the location of the original trouble. The advantage of the reactor in this respect is, of course, a fundamental one from the standpoint of power operation. It is in general of proportionate importance from the inductive interference point of view, at least where a power network is involved in parallels with communication circuits at several places, as is not infrequenctly the case near large cities. A breakdown to ground in the power network on a different phase from that originally involved, and in a different locality, may lead to large phase-to-phase currents in the earth, from the second fault to the first, or to a ground intentionally placed on the phase first involved, in order to short circuit the arc. The inductive effects thus become electromagnetic in character, and the interference produced in this manner may be severe.

The possibility that the reactor might tend to produce a greater

[3] "Electrically" is used here and elsewhere in this paper in the sense in which "electrostatically" is perhaps more commonly used. The phenomena involved are not static and the latter word is inappropriate on this account.

overvoltage on a sound phase at the instant of grounding than would be the case in an isolated system is, of course, of importance in this connection. This has been examined from a theoretical standpoint in the second subdivision of this section, with the conclusion that there is no material difference in this respect.

There remains the method of grounding in which a high resistance is employed in the connection from neutral to ground. By "high" here may be meant the "critical"[4] resistance or one of smaller magnitude, but still so large that in the event of a solidly grounded phase, the sound phases are brought to subtsantially full delta voltage above ground. There are probably few cases where electromagnetic indutive effects due to accidental grounds on a power system are a matter of importance, in which a neutral resistance small enough to avoid this rise of voltage on the sound phases would be effective as a measure of relief. This method of grounding would thus not avoid the electrically induced voltages which arise when the reactor is used, although it would presumably be effective in preventing the spread of trouble to other parts of the power system if positive operation of selective relays is secured. Inasmuch as it presents fewer difficulties from this last point of view than the Petersen reactor, grounding through a moderate resistance has a definite advantage over the latter method from the standpoint of inductive effects at fundamental frequency, provided sufficient resistance can be used to limit the electromagnetically induced voltages to tolerable values.

Where this is impracticable from the standpoint of power system operation, the relative merits of the two systems would have to be decided by balancing the effective suppression of the transient electromagnetic inductive effects by means of the reactor, plus the expectation of occasional disturbances continuing over the intervals necessary for the location and disconnection of the faulty line, against the imperfect suppression of the former effects by means of the resistor, plus the limitation to very brief intervals of electrically induced disturbances otherwise the same as with the reactor. It is obvious that the factors controlling such a decision would vary widely in different cases, and that practical experience with both methods would be of great value in estimating their relative importance. It is, of course, possible that future development may remove some of the disadvantage at which the Petersen coil now finds itself in respect to the matter of relay protection for interconnected networks. Such development would presumably be of importance also to the critical

[4] *I.e.*, the resistance for which a discharge to ground passes from the oscillatory to the non-oscillatory type.

resistance which, as a method of grounding the neutral, would generally suffice to prevent interference from ground currents at times of faults to ground, but which apparently presents difficulties from the relay standpoint similar to those involved in the use of the Petersen coil.

II. Effects with Power System in Normal Condition

1. *Fundamental Frequency*

Referring to Fig. 5 (we now take account of inequalities in the admittances to ground),

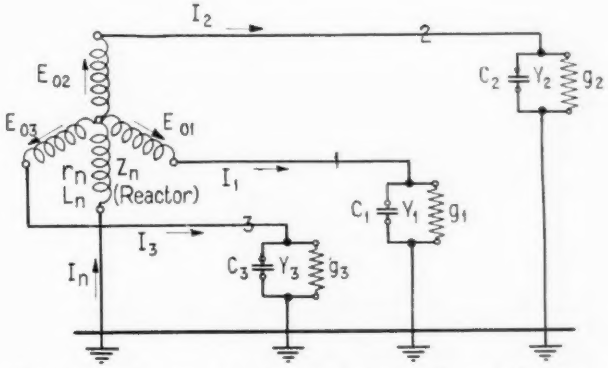


Fig. 5—Three-phase System with Neutral Grounded Through Reactor. Admittances to Ground Not Balanced.

$$E_{01} - \frac{I_1}{Y_1} = E_{02} - \frac{I_2}{Y_2} = E_{03} - \frac{I_3}{Y_3} = Z_n I_n,$$

$$I_1 + I_2 + I_3 = I_n,$$

so that

$$I_n = \frac{Y_1 E_{01} + Y_2 E_{02} + Y_3 E_{03}}{1 + Z_n Y},$$

where $Y = Y_1 + Y_2 + Y_3$ = total direct admittance to ground. If the impressed voltages are balanced

$$I_n = \frac{E_{01}}{1 + Z_n Y}(Y_1 + Y_2 e^{j120°} + Y_3 e^{j240°}).$$

The parenthesis is the "residual admittance" [5] to ground and, if the three leakances to ground are equal, it can vanish only if

[5] Inductive Interference between Power and Communication Circuits, California Railroad Commission, p. 269.

the three direct capacities to ground are the same. The equation is equivalent to

$$I_n = \frac{E_{rc}}{3} \frac{Y}{1 + Z_n Y},$$

where E_{rc} is the "characteristic residual voltage"[6] of the (isolated) system and is equal in magnitude to 3 E_{01} times the ratio of the residual admittance to ground to the total admittance to ground.

We have

$$YZ_n = r_n g - \omega^2 L_n C + j\omega\,(Cr_n + L_n g)$$

r_n and L_n being the resistance and inductance of the earth coil, and g and C the total leakance and capacity to ground, respectively. Also, for resonance

$$\omega^2 L_n C = 1$$

and hence the above expression for I_n becomes (at fundamental frequency)

$$I_n = \frac{E_{01}\,(Y_1 + Y_2 e^{j120°} + Y_3 e^{240°})}{\frac{r_n}{\omega L_n}\frac{g}{\omega C} + j\left[\frac{r_n}{\omega L_n} + \frac{g}{\omega C}\right]}$$

if the earth coil is adjusted for accurate resonance.

If in this equation the denominator on the right be denoted by x and the fractional unbalance of the admittance to ground by y $\left(i.e., y = \frac{Y_1 + Y_2 e^{j120°} + Y_3 e^{j240°}}{Y}\right)$, then

$$I_n = \frac{yYE_{01}}{x} \tag{3}$$

and, V_n being voltage between neutral and ground,

$$\begin{aligned} V_n &= Z_n I_n \\ &= \frac{y\,(x - 1)}{x} E_{01}. \end{aligned} \tag{4}$$

If the losses in the system (including the earth coil) are small, x is small compared to 1. Thus we get, for the absolute value of V

$$|\,V_n\,| = \left|\,\frac{y}{x}\,.\,E_{01}\,\right|, \text{ approximately}$$

and the absolute value of the residual voltage is three times this, approximately.

[6] *Ibid.*, p. 257.

In substance, this means that, at fundamental frequency and with small losses, the fractional admittance unbalance should be kept small compared to the ratio of the resistance to the reactance of the coil, if unduly high voltages to ground are to be avoided. (It is supposed that $g/\omega C$ is of the order of $r_n/\omega L_n$, or smaller—a condition probably satisfied even with the best practiable design of coil, except under very wet line conditions.) The point involved here is, of course, an important one from the standpoint of power system operation. It is also important from the standpoint of electrically induced voltages in exposed communication circuits. The admittance unbalance can be kept within the necessary limits by suitable power circuit transpositions.

The absolute magnitude of the fundamental frequency neutral current is obtained from the expression for $|V_n|$ by dividing by the coil impedance (or directly from equation (3), and is, approximately,

$$|I_n| = \frac{|yE_{01}|}{r_n + \frac{g}{\omega C}\omega L_n}.$$

For a system with dead-grounded neutral the fundamental frequency residual current is $yE_{01}Y$ and is thus smaller than that just found for the case of the reactor in the ratio of $|x|$ to 1, approximately. It is evident, however, that the magnitude of the neutral current with reactor is controllable by means of transpositions, as in the case of the neutral and residual voltages. The inductive effects of this current should be of small consequence with an amount of transposing sufficient to keep the line voltages to ground within limits desirable from the standpoint of power system operation.

2. Harmonics

In the following discussion, we retain the assumption of lumped constants, so that the results are not applicable to extensive networks without modification.

With this restriction, at harmonic frequencies other than the third or one of its odd multiples, the above approximate equation for V_n becomes

$$V_n = \frac{y'(x'-1)}{-1 + \frac{1}{m^2} + x'} \cdot E_{01}, \text{ approximately,}$$

m being the order and E_{01} the voltage of the harmonic and x and y accented to denote that they are to be taken for the frequency in

question. For small losses x' may be neglected in comparison with unity, as before, giving

$$|V_n| = \frac{|y' E_{01}|}{1 - \dfrac{1}{m^2}}, \text{ approximately.} \tag{5}$$

For isolated neutral

$$|V_n| = |y' E_{01}|. \tag{6}$$

Thus, even for the fifth harmonic, the right hand side of (5) is only 4 per cent. in excess of the value it would have if the neutral were isolated.

For harmonics whose orders are not divisible by three the residual voltage is three times V_n. Thus from the standpoint of noise interference from voltages, a system grounded through a Petersen coil behaves practically as though the neutral were isolated, so far as these harmonics are concerned. As with the fundamental, power circuit transpositions are available for the reduction of residual voltages of these frequencies.

Residual currents of frequencies belonging to this series of harmonics, which are not present at the ends of the line with isolated neutral, are introduced by grounding through the reactor, but they are of minor importance, as may be judged by comparing the neutral currents with the reactor and with dead grounded neutral. With the reactor, the neutral current of a harmonic of order m not a multiple of 3 is found from (5) to be, in absolute value,

$$|I_n| = \frac{m\,|y' E_{01}|}{(m^2 - 1)\,|Z_n|}, \text{ approximately,}$$

Z_n being the coil impedance at fundamental frequency, while with dead-grounded neutral, it would be

$$|I_n| = |E_{01} y' m Y|, \text{ approximately,}$$

in which Y is the total admittance to ground at fundamental frequency. Thus the magnitude of the neutral current with the reactor is approximately $1/(m^2 - 1)$ of its magnitude with dead-grounded neutral. The noise effects of residual currents of these magnitudes will generally be insignificant compared to those arising from other sources, particularly if the power circuit capacities to ground are well balanced.

For the third harmonic, or one of its odd multiples, we get

$$V_n = Z'_n I_n = \frac{E_{01} Z'_n Y'}{1 + Z'_n Y'},$$

in which the symbols for coil impedance and line admittance are accented to denote that they refer to the harmonic frequency in question;

$$V_n = \frac{E_{01}m^2\,(x' - 1)}{1 + m^2\,(x' - 1)};$$

and

$$|\,V_n\,| = \frac{|\,E_{01}\,|}{1 - \dfrac{1}{m^2}}, \text{ approximately.}$$

For isolated neutral,

$$V_n = E_{01}$$

The neutral is thus subjected to a third harmonic voltage some 12 per cent. greater than if it were isolated, but for the higher harmonics belonging to this series (of the third and its odd multiples), the difference is inappreciable.

The residual voltage for a harmonic of this series is

$$V_r = 3\,(E_{01} - V_n) \tag{7}$$

$$= 3E_{01}\,\frac{1}{1 + m^2\,(x' - 1)};$$

and

$$|\,V_r\,| = \frac{3\,|\,E_{01}\,|}{m^2 - 1}, \text{ approximately.} \tag{8}$$

The corresponding neutral current is

$$I_n = \frac{|\,E_{01}\,Y'\,|}{1 + Z'_n Y'}$$

and

$$|\,I_n\,| = \frac{|\,E_{01} Y'\,|}{m^2 - 1}, \text{ approximately.} \tag{9}$$

From the standpoint of noise interference in telephone circuits, residuals of the series consisting of the third harmonic and its odd multiples are frequently troublesome where the neutral of a three-phase system is grounded directly or through a low resistance. These residuals, of course, are not affected by power circuit transpositions, either as to their magnitudes or as to their inductive effects upon exposed telephone circuits. It is therefore of interest to examine the expressions just obtained for the case in which the neutral is grounded through the coil. While the neutral current will not be the same as

the residual current except when only one line is supplied from the transformer bank, the effect upon the former should in general be at least approximately proportional to the effect upon the residual current in any line supplied from the bank.

In writing equation (7), any difference between the induced voltage E_{01} and the voltage appearing between line and neutral has been ignored. To the extent that this is justifiable, the expressions for the case of the solidly grounded neutral may be obtained by making the denominators of the right hand sides of (8) and (9) each unity. If the transformer bank is provided with a delta winding of low impedance, in particular if it is connected delta on one side, this procedure gives a fair approximation to the correct expressions, since the impedance through which the voltage E_{01} regulates is in this case merely the transformer leakage impedance. The resulting conclusions with respect to the advantage of the reactor—for example, that the third harmonic residual voltage or neutral current is 1/8 as large, the ninth 1/80 as large, etc., with the reactor as with solidly grounded neutral—should not, in any event, be unfavorable to the latter method of grounding unless the electrical length of the line approaches the point at which its reactance to ground becomes positive.

If the transformers are so connected as to provide no path for triple harmonic magnetizing currents other than through line admittance to ground and the impedance between neutral and ground, the induced voltage E_{01} is not the same for the two methods of grounding under consideration, because the impedance to the triple harmonic magnetizing currents is appreciably different in two cases. A convenient method of taking this effect into account is to regard the induced voltage as due to a fictitious impedanceless generator of determinate voltage regulating through the mutual impedance of the transformer windings for the frequency in question.[7] If Z'_m is one-third of this mutual impedance and V_{01}[8] is the voltage of the fictitious generator, the expression for the neutral current with ground connection through the reactor becomes

$$I_n = \frac{V_{01}Y'}{1 - m^2 + Y'Z'_m}, \text{ approximately,} \tag{10}$$

[7] H. S. Osborne, Trans. A. I. E. E. 34, p. 2175, 1915.

[8] The voltage thus assumed is, of course, not identical with the induced voltage for which the symbol E_{01} has hitherto been used, and for this reason the new symbol V_{01} is used for it. The corresponding E_{01} would be V_{01} diminished by the drop through the mutual impedance.

and the residual voltage is

$$V_r = 3\,(V_{01} - I_n Z'_m - V_n)$$

$$= \frac{3\,V_{01}}{1 - m^2 + Y'\,Z'_m} \text{ approximately.} \qquad (11)$$

The corresponding expressions for solidly grounded neutral are obtained by omitting m^2 in the denominator for each of the equations just derived. Thus the advantage of grounding through the reactor relative to grounding directly depends on the magnitude of $Y'Z'_m$ as compared to the square of the order of the harmonic. Z'_m depends upon the voltage and the kva. capacity of the transformers and is mostly inductive reactance. For high voltage transformer banks of small capacity feeding very extensive networks, the gain indicated by equations (10) and (11) from the use of the reactor would probably not be large. It would be important, however, where the aggregate capacity of the supply transformers is moderate or large and the connected network is of moderate extent and voltage. For instance, using the data of the example considered in an earlier part of this paper and taking $Z'_m = jL'_m\omega' = \frac{1}{3} \cdot j\ 9{,}000$ as an appropriate value for a total transformer capacity of 7,000 to 8,000 kva., with line voltage from 20,000 to 30,000, we should have $L'_m C\omega'^2$ equal to about 4 at 180 cycles/sec. In other words, in this case, the employment of the reactor would reduce the residual voltage and the neutral current of the third harmonic frequency due to a star-star solidly grounded transformer bank by about 75 per cent., and residuals of other frequencies belonging to the same series probably by larger amounts.

In the earlier discussion relating to harmonics not belonging to the triple series, comparison was made between a system grounded through a Petersen reactor and the isolated system. In a similar comparison with respect to the triple harmonic series, the isolated system has the advantage, since residuals of this series theoretically do not appear in such a system, as the voltages are not impressed between wires. As a practical matter, an isolated system would probably not be entirely free of triple harmonic residuals, owing to dissimilarities in transformers or elsewhere. Such accidental effects can hardly be taken into account in a theoretical discussion. However, in setting up a comparison between the isolated system and that grounded through the reactor, an idea of the relative importance of the triple harmonic residual voltages existing in the latter case can perhaps be obtained by comparing their theoretical magnitudes with the theoreti-

cal magnitudes of residual voltages in the isolated system of non-triple frequencies.

The residual voltage due to one of these non-triple frequencies, which is three times the neutral voltage, is $3y' E'_{01}$, according to equation (6). Here y' is the fractional residual admittance and E'_{01} may be taken as the induced voltage in the transformer for the frequency in question. For a harmonic belonging to the triple series, with neutral grounded through the reactor, the absolute value of the residual voltage is $\dfrac{3 \mid E''_{01} \mid}{m^2 - 1}$ (equation (8)) m being the order of the harmonic and E''_{01} the induced voltage, if we assume the transformer bank provided with a low impedance path for triple harmonics, and therefore neglect the difference between the induced and the terminal voltages. The ratio of the triple-series residual voltage to the other is thus, in absolute value, $\dfrac{\mid E''_{01} \mid}{(m^2 - 1) \mid y' E'_{01} \mid}$.

If we take the ninth as the harmonic of the triple series and assume equal values of the induced voltages E''_{01} and E'_{01} it will be seen that $\mid y' \mid$ must be of the order of 0.01 if the residual voltage of the triple harmonic series is to be as large as the other. This amount of unbalance is somewhat larger than has been found at this frequency (540 cycles/sec.) in an actual transposed line.[9] If we consider the higher harmonics of the triple series, $\mid y' \mid$ would have to be made progressively smaller in order that the ratio might remain unity. Thus, for the 21st harmonic, $\mid y' \mid$ would have to be of the order of 0.002. While, of course, $\mid y' \mid$ may be made as small as desired by sufficiently close power circuit transpositions, it appears that in practical cases where transformer banks have delta windings, one may expect the residual voltages of the triple series, introduced by changing from an isolated system to one grounded through the reactor, to be relatively unimportant except in the case of the third harmonic and perhaps in that of the ninth. This statement would not be true if, as with star-star transformers under some circumstances, no low impedance path is provided for magnetizing currents of the triple harmonic series. Such cases are not common in operating practice.

The method of estimating comparative effects here applied to the case of triple harmonic residual voltages is not available for residual currents. To take account of the latter in comparing the isolated

[9] Inductive Interference between Power and Communication Circuits, California Railroad Commission, Technical Report No. 51.

system and the system with neutral grounded through the reactor, recourse may be had to the indirect method of reference to the solidly grounded neutral system, as in the discussion of residual currents of the non-triple series on page 52. Such a procedure, of course, involves a reference to general experience also. It has been shown in the earlier discussion that for a triple series harmonic of order m the neutral current with the reactor is approximately $1/(m^2 - 1)$ as large as with the dead-grounded neutral if a low impedance path for triple frequency magnetizing currents is provided, as by a delta winding. The establishment of this system of neutral currents, even though they are small, when a previously isolated system is grounded through a Petersen reactor, constitutes an addition to the residuals which produce induction in neighboring circuits. However, it is not to be expected that the added inductive effects would be important. Where no low impedance path for the triple series magnetizing currents exists, the reactor is relatively less effective in suppressing residual currents of this series. The triple harmonic neutral currents of a power system connected in this manner and grounded through a Petersen coil might in some cases lead to inductive effects of some significance.

In general, for harmonics of orders not divisible by three, grounding through a moderate resistance (large, however, compared to other impedances involved in a short circuit to ground) will be more advantageous as regards residual voltages, and less advantageous as regards residual currents, than grounding through the reactor. Grounding through zero impedance would, of course, generally lead to the smallest residual voltages and the largest residual currents of these frequencies. For frequencies belonging to the triple series, grounding through the reactor will be considerably more advantageous than grounding through a moderate resistance as regards both residual voltages and residual currents. It may be expected that with moderate neutral resistance, residual currents and voltages of the triple series will both be nearer in magnitude to those obtaining with zero neutral impedance than to those obtaining with the Petersen coil. The moderate neutral resistance is relatively more effective at the higher frequencies in reducing residual currents of all harmonics and residual voltages of the triple series; for harmonic residual voltages not belonging to the triple series, it is relatively more effective at the lower frequencies.

I wish to express my gratitude for helpful suggestions and criticism received in the preparation of this paper from Messrs. L. P. Ferris and R. G. McCurdy, and also from Mr. R. K. Honaman.

SUMMARY

1. At times of a fault to ground on a power system with neutral grounded through a Petersen reactor, the action of the latter tends to extinguish the arc and to prevent its restriking. Theoretical considerations, applied to a practical case, indicate that the transient over-voltage on a sound phase at the instant of occurrence of the fault is substantially the same as in a system with isolated neutral.

2. Grounding the neutral through a Petersen earth coil instead of directly or through a low resistance would largely prevent the electromagnetic inductive effects to which exposed communication circuits are liable at times of faults to ground in systems grounded in the latter manner. (Extensive high voltage networks are perhaps an exception to this statement. But even here, the electromagnetic inductive effects would in general not be greater with the reactor than with isolated neutral.) However, effects due to electric induction similar to those from an isolated system may be expected to appear. Except for long, close parallels involving open-wire communication circuits these effects should in general be much less severe than the electromagnetic inductive effects from a system with dead-grounded neutral. The extent and severity of the inductive effects experienced from the system grounded through the reactor would further tend to be smaller than with the isolated system, because of the effect of the reactor in preventing arcing grounds.

3. Grounding the neutral through a resistance large compared to other impedances involved in a short circuit to ground should have an advantage over grounding through the Petersen reactor, in that the former method presents fewer difficulties in respect to power system protective relays, so that it would reduce the possibility of the continuance of inductive disturbances over considerable periods of time, which might be involved in grounding through the reactor, under present relay practice. From an inductive interference standpoint, a choice between the two methods would depend upon the circumstances of particular cases. Advances in the art of relay protection would improve the position of the reactor in such considerations.

4. Under normal power system operating conditions, the use of the reactor may lead to excessive residual voltages of fundamental frequency if the admittances from phases to ground are unbalanced. Such unbalance may be reduced to the extent necessary from this point of view by power circuit transpositions.

5. Under normal operating conditions, it is to be expected that the residual voltages and currents of the triple harmonic series occurring

with neutral grounded through zero or a low impedance would be largely reduced by grounding through the Petersen reactor instead. Residuals of other harmonic frequencies should be substantially the same as with isolated neutral, and are controllable by means of power circuit transpositions. The method of grounding the neutral through a resistance of moderate value is favorable to the reduction of residual voltages of the harmonics whose orders are not multiples of three, but is relatively unfavorable to the suppression of residual currents of these frequencies. It is also considerably less effective than the reactor in preventing residuals, either voltages or currents, belonging to the triple harmonic series, which are not amenable to treatment by transpositions.

Philadelphia-Pittsburgh Section of the New York-Chicago Cable[1]

By JAMES J. PILLIOD

SYNOPSIS: Engineering and construction features involved in a complete telephone cable system over 300 miles in length and connecting Philadelphia and Pittsburgh, Pa., are described in the following paper. This cable is designed to operate as an extension of the Boston-Washington underground cable system with which it connects at Philadelphia. It is also designed for operation in connection with the Pittsburgh-Chicago cable now under construction, and other cable projects included in a comprehensive fundamental plan.

Beginning with the fundamental factor of public requirements for communication service between cities separated by various distances, there are next considered the methods available to provide this service. Small-gage, quadded, aerial cable, which was decided upon for use in this section after careful economic studies, is described in a general way and the important advantages of the application of loading and telephone repeaters are outlined. The use, in connection with this cable, of the recently developed metallic telegraph system for cables is referred to and some facts are given regarding power plants, test boards and buildings. A few of the many possible combinations of cable and equipment facilities into complete telephone circuits, which will furnish the service required as economically as now possible, are illustrated.

The necessity of complete coordination of the many factors involved in a project of this kind is emphasized.

INTRODUCTION

THE placing in service in the latter part of 1921 of the final section of a continuous telephone cable over 300 miles in length between Philadelphia and Pittsburgh marked a new point of achievement in the steady development and construction of facilities designed to render to the public the best possible long-distance telephone service. Furthermore, this cable forms an important part of a comprehensive plan of long-distance cable construction throughout that section of the United States lying in general east of the Mississippi River and north of the Ohio and Potomac Rivers.

In the discussion of a project of this kind which involves many new practices and the expenditure of several millions of dollars and which, with related work already completed, forms the groundwork for large expenditures in the future, it is usual to inquire first into the underlying reasons for carrying out the project and then into the methods adopted. In the following discussion an endeavor will therefore be made to furnish some information on these two items in their relation to the Philadelphia-Pittsburgh cable, although, as will be obvious, the many different points can be covered in only

[1] Presented at a meeting of the Philadelphia Section of the A. I. E. E., January 9, 1922, presented at the Annual Convention of the A. I. E. E., Niagara Falls, Ont., June 26–30, 1922, and appearing in the Journal of the A. I. E. E. for August, 1922.

the most general way in the space available. However, before going ahead with the discussion, I would like to point out that this project is not unlike many others in that, as a whole and in the component parts, there have been required, first, the careful consideration and decisions of the executives, then the underlying work of many scientists, inventors and engineers, then the skilled work of the manufacturers and construction forces, and finally the maintenance and operation by trained people who are responsible for the continuous service so vitally necessary to the industrial and social structure of the country. The point to be emphasized here is that the coordination of all of these factors and the close cooperation of all of the many hundreds of people concerned are the important things.

General Cable Plans and Routes

Fig. 1 is an outline map of a section of the United States and shows the routes of existing and proposed long telephone cables of the Bell system. It will be noted that the present and proposed routes follow in a general way the routes of trunk-line railroads. This general section contains more than 50 per cent of the entire population of the United States but less than 15 per cent of the area, and the industrial and telephone development is, of course, very great. Furthermore, the nearby surrounding states, supplying as they do large quantities of food products and raw materials, are commercially related to this section in a very peculiar way and this fact greatly influences the long-distance telephone development along the particular cable routes indicated. The routes through the State of Pennsylvania and the offices at Philadelphia and Pittsburgh, which are the terminals of the cable that is more particularly the subject of this discussion, occupy strategic positions in this system.

Circuits of the American Telephone and Telegraph Company and the Bell Telephone Company of Pennsylvania are carried over these routes and this cable was jointly planned and installed by these companies.

Fig. 2 is an outline map of the State of Pennsylvania and shows the situation in this section a little more in detail. On this map are shown some of the larger cities and routes of the longer and more important toll and long-distance telephone lines. As indicated, these lines are mainly of the familiar aerial wire type which has been generally used in the past for this purpose and which is today the most efficient and economical type of construction for many cases. In the general section between Philadelphia and Pittsburgh the

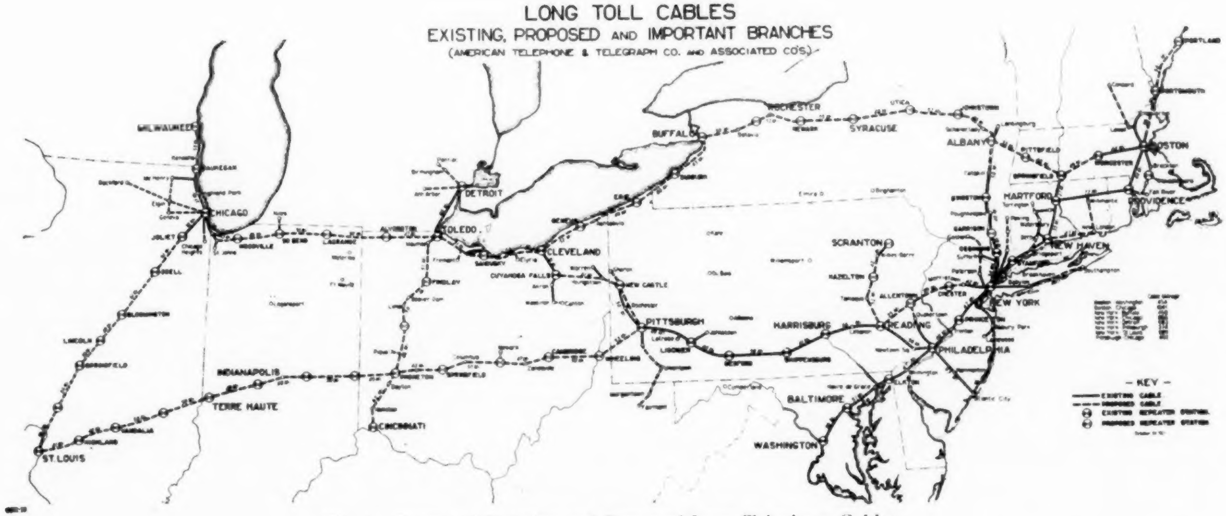


Fig. 1—Routes of Existing and Proposed Long Telephone Cables

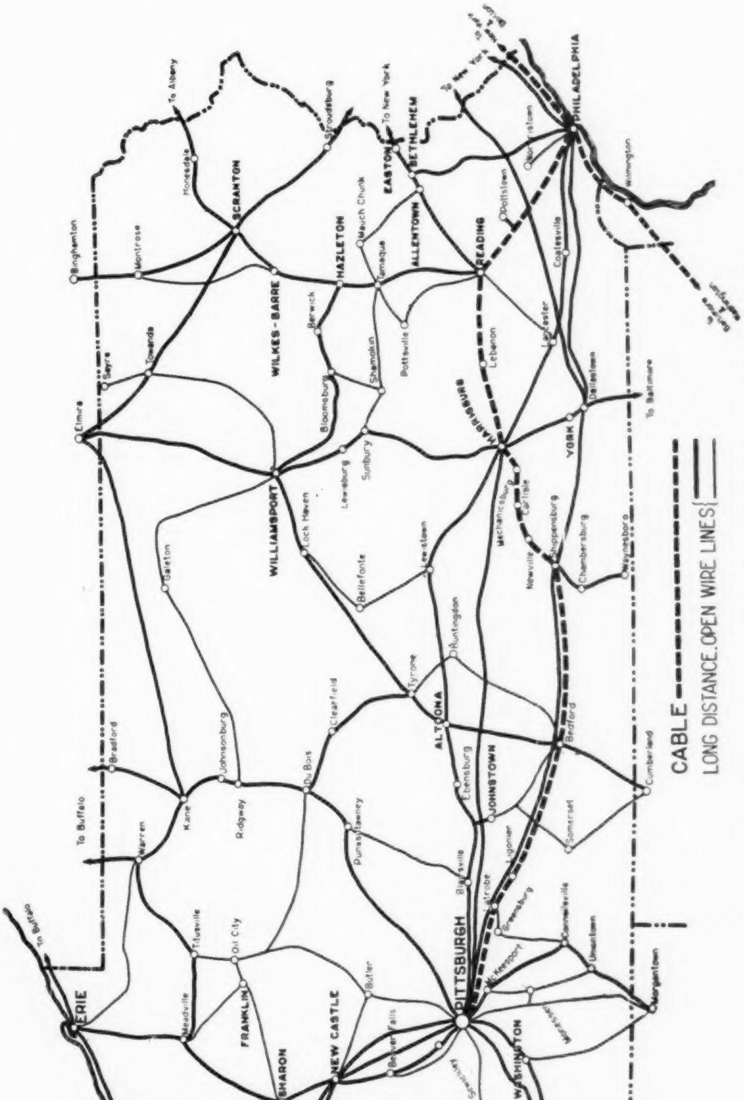


Fig. 2—Outline Map of Pennsylvania, Showing Aerial Line and Cable Routes

requirements for circuits are very heavy and in addition, as is well-known, the topography of the country is such that the through routes which can economically be used for pole lines are limited. At present, these few routes are fully occupied by the pole lines of the various utilities and included in these lines are three fully loaded telephone trunk lines. Another item of importance in the consideration of aerial wire construction is the very severe damage frequently experienced in many sections of the country on heavy aerial wire lines from ice and wind storms. Even lines built with exceptional strength fail in these storms and the interruptions to service are serious matters to the users as well as to the telephone companies. The restoration costs under the conditions that naturally exist at such times are abnormally high.

Fig. 3—Damage to Section of New York-Boston Main Line Near Worcester, Mass. Storm of November 28, 1921

Figs. 3 and 4 show the effects at one point of the ice and wind storm in New England on November 28, 1921, and are proof that this problem is real. This particular spot is near Worcester, Mass., and the line is a section of one of the principal aerial wire routes between New York and Boston. In this storm, many thousands of poles were broken and even where a few poles remained standing due to specially strong construction, the load of ice combined with the wind was too great for the wires to withstand. There is therefore a practical limit to the number of wires that can be safely and economically carried on a pole line.

Where the practicable routes for pole lines are limited, where the

existing pole lines are fully loaded, and where estimated future circuit requirements are of considerable magnitude, it is obvious that different methods of providing facilities, if available, must sooner or later be given serious consideration. The conditions between Philadelphia and Pittsburgh and in general along all of the cable routes shown on Fig. 1 are now, or are expected within a few years to be, such as to make the use of some type of construction other than aerial wire desirable for most of the circuits.

After careful studies of the circuit requirements for future periods

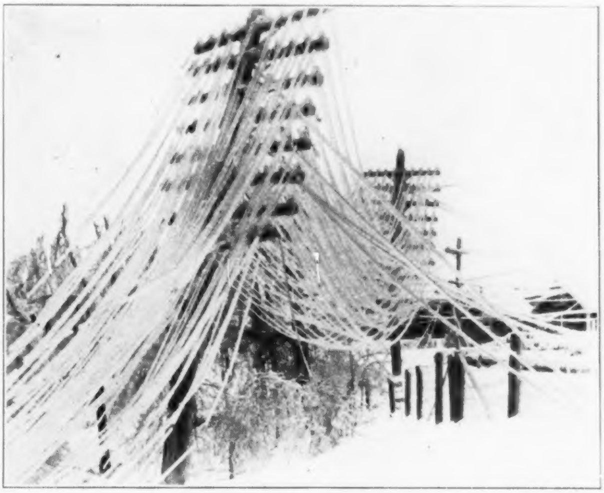


Fig. 4—Section of New York-Boston Main Line Showing Wires Heavily Loaded with Ice. November 28, 1921

and of the methods available for providing long-distance telephone facilities, which in general are aerial wire and cable, it has been decided that for relief in these sections the cable method will give the best and most economical results. Long underground cables, as is well-known, have been in operation for many years between Boston, New York, Philadelphia, Baltimore and Washington, Chicago and Milwaukee and in other sections. However, the type of cable and associated apparatus which is now being used in the development of the more comprehensive plan is quite different from that originally used between Boston and Washington and in the other sections, particularly in the use of copper conductors of a smaller gage combined with improved loading coils, the vacuum tube telephone repeater

and other methods and apparatus which are the result of recent developments. Lead-covered aerial cable supported on wooden pole lines is to be used in general on all of the routes except in the two sections just mentioned and through cities or where special conditions exist for short distances. The possibility of now using conductors of No. 16 and No. 19 A. W. G. instead of conductors up to

Fig. 5—General View of Pole Line Carrying Aerial Cable

No. 10 A. W. G. as in the older cables, has contributed to make aerial construction rather than underground conduit the more economical in many sections, as one cable will provide for a much greater number of circuits and consequently fewer cables will be required.

Line Construction

The general type of aerial construction which was used for over 250 miles of the total distance of 302 miles from Philadelphia to Pittsburgh may be seen from Figs. 5 and 6 which illustrate the poles, steel suspension strand, metal supporting rings and the cable. The poles are 25-foot untreated chestnut spaced 100 feet apart and designed to carry additional cables in the future. While the poles are new and carry only one cable they have a factor of safety of about 9 under the most severe storm conditions expected, but this will, of course, be reduced as other cables are placed and will gradually be decreased on account of decay at the ground line until it becomes necessary to start replacing the poles. Many of these poles were grown near the locations where they now stand. In other sections, it is planned

to use butt-treated chestnut or cedar poles, or creosoted pine poles where these prove to be the more economical.

The galvanized steel suspension strand has a breaking strength of about 16,000 pounds and the actual tension under normal conditions is about 7,000 pounds. In placing the strand, it is necessary to pull it to just the right tension in order that when the cable is hung it will have the proper sag. The correct tension is readily determined by what is known as the "oscillation" method. The metal rings are spaced 16 inches apart and the cable weighs about 7½ pounds per foot.

The size and make-up of the cable vary somewhat with the number of circuits of the various types that are to be provided in the different sections, but in general it is full size, that is, its over-all diameter is

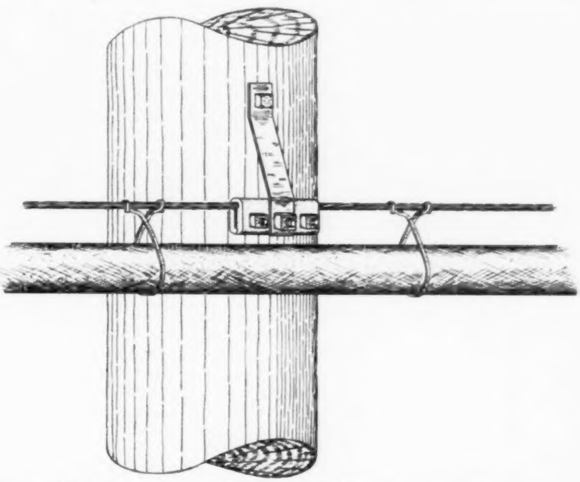


Fig. 6—Method of Supporting Aerial Cable and Messenger

2⅝ in. which is about the maximum size of telephone cable. The sheath is of lead-antimony alloy, one-eighth of an inch thick, and under normal conditions it is, of course, air-tight to keep moisture from entering. The cable for the aerial section was received from the factory in 500-foot lengths, this being largely determined by the arrangement necessary to permit the proper installation tests.

Route

We might next consider the route selected and for this purpose Fig. 2 will again be helpful. It will be noted that starting at Phila-

delphia, the cable is routed to Reading touching Pottstown, Phoenixville and other points. From Reading to Harrisburg the cable follows closely the William Penn Highway, although in sections it was necessary to obtain private right-of-way or to use longer routes removed from this highway on account of the lines of various kinds already in operation there. It is very desirable for economic reasons to keep the length of these cables as short as possible and in some cases this is absolutely necessary to obtain proper operating conditions.

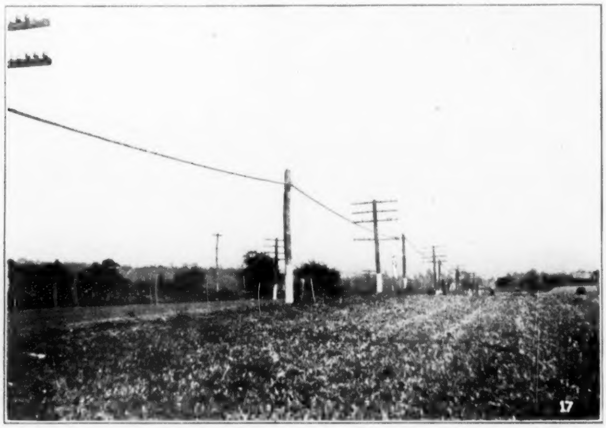


Fig. 7—Cable Line on Seven-Mile Stretch of Lincoln Highway. Aerial wire line to be dismantled later.

However, the most direct routes cannot always be used, for many obvious reasons, and this problem required careful consideration in all sections of the cable.

Between Harrisburg and Pittsburgh the Allegheny Mountains had to be crossed and for this crossing only two general routes were found practicable, the first following an existing pole line which is the New York-Chicago telephone line through Lewiston, Altoona, etc., and which we may call the northern route, and second a southern route through Shippensburg, Bedford and Ligonier for the most part along the Philadelphia-Chicago line and also the Lincoln Highway. A middle route which is now used for the Harrisburg-Pittsburgh line was not seriously considered as the country was too rough for economical construction and maintenance and no important advantages were to be obtained. After careful surveys and cost studies, taking into account all existing and anticipated conditions, such as circuit

requirements and towns to be reached, length of practicable routes, maintenance conditions, freedom from probable physical and electrical interference, etc., it was decided to build on the southern route.

This route, while of nearly the same length as the northern one and offering some important advantages, was not free from difficulties as it crosses the Allegheny Mountains within a few miles of the highest

Fig. 8—Cable Line Across Valley at Grand View

point. Fig. 7 shows the cable line on what is known as the seven-mile stretch of the Lincoln Highway east of Ligonier, and here the going was fairly good. The Philadelphia-Chicago aerial wire line is also shown and two of the crossarms carrying 10 wires each are to be removed in the near future and the circuits operated in the cable. It is planned to remove the remaining two crossarms later on. Fig. 8 shows the cable across a valley and is taken from the point on the Lincoln Highway called Grand View. Fig. 9 shows the crossing of the Juniata River east of Bedford where special construction was used. Fig. 10 shows just one example of the conditions encountered in crossing the many mountains and a photograph does not do the scenery or the construction difficulties justice. On account of the steep slopes, clamps are used at many points to fasten the cable to the strand.

Narrow-gage timber railroads were used in the mountains where possible to get material to the job and Fig. 11 shows one of the regular

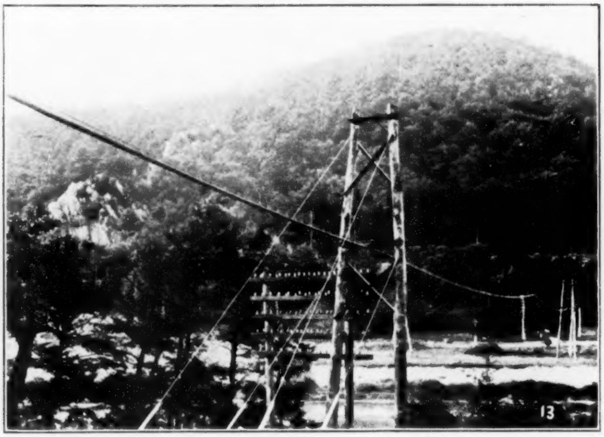


Fig. 9—Cable Crossing at Juniata River

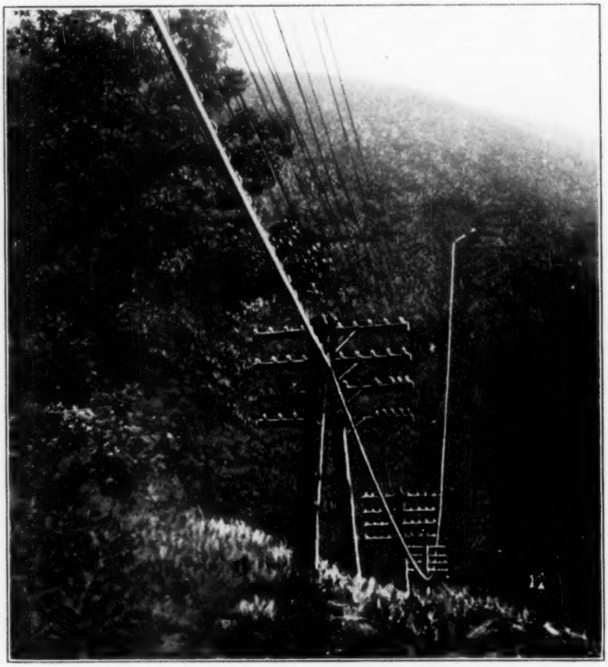


Fig 10—Cable Line on Steep Slopes

flat cars adapted for our purpose. Fig. 12 shows two 5-ton tractors in action on top of one of the mountains. As many sections of the country are very rough and highways several miles distant it seemed that no other method of transporting the cable reels, which weigh

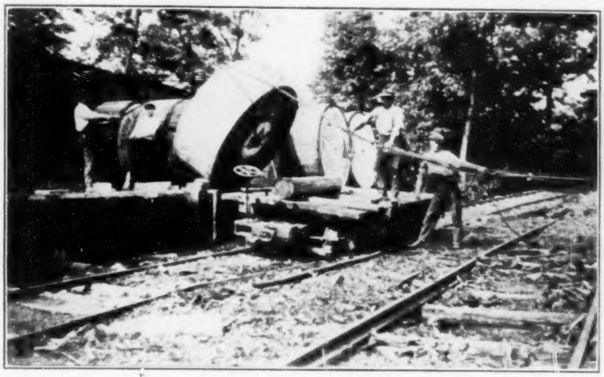


Fig. 11—Narrow-Gage Mountain Railroad and Flat Car

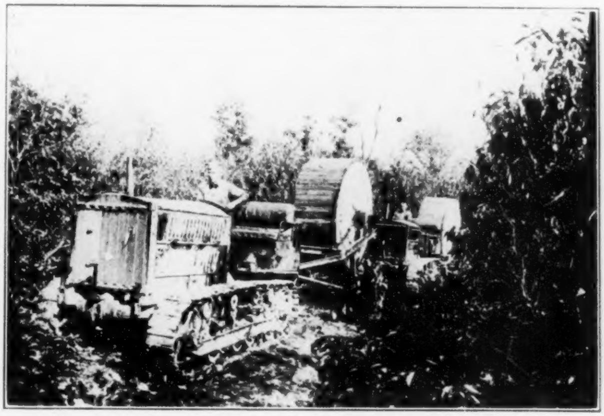


Fig. 12—Tractors Placing Cable Reels in Rough Country

nearly 5,000 pounds, could possibly be used, and certainly no other means would have been as satisfactory. Even with these methods the cable reels could not always be delivered where desired and in some cases it was necessary to pull the sections of cable through the rings for a distance of nearly a mile to get them in place.

Cable Make-Up

As stated before, the make-up of the cable varies somewhat with the circuit requirements in the different sections but the wires and arrangement in a typical section of cable are roughly illustrated in Fig. 13.

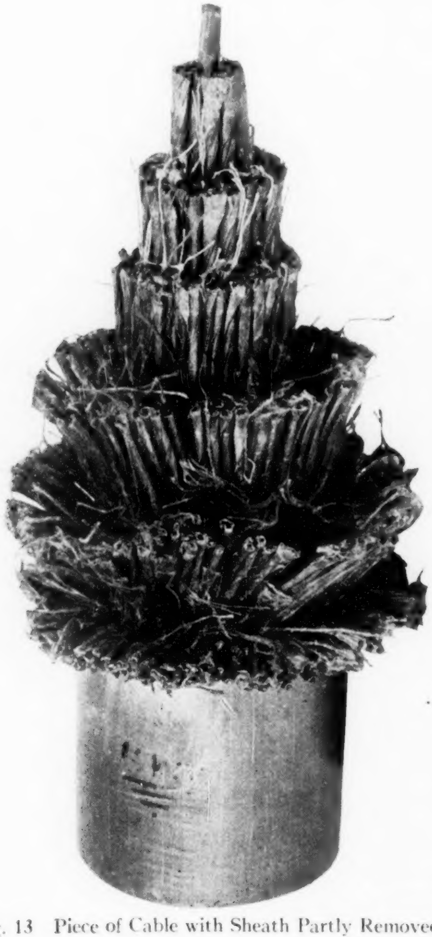


Fig. 13 Piece of Cable with Sheath Partly Removed

The cable is of quadded construction, that is, the wires are first wrapped with dry paper for insulation and twisted into pairs and then two pairs are twisted into what is called a quad. These quads are

arranged in concentric layers as shown and great care and skill are required in the design and manufacture or there is certain to be serious cross-talk between the several hundred circuits when used for long-distance service. Even after the application of the best present manufacturing methods, tests are made on all circuits at three points in each loading section of 6000 feet while the cable is being spliced. These tests are made in order to determine the best possible arrange-

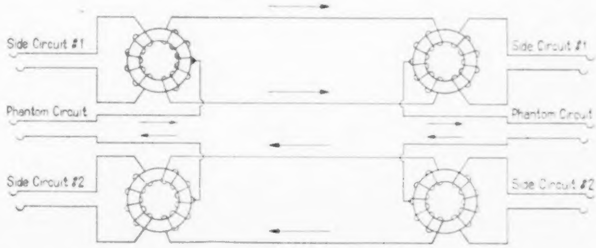


Fig. 14—General Phantom Circuit Arrangement. Four wires providing three circuits

ment of conductors for still further reducing cross-talk between circuits, and the splicing is done accordingly.

There are 19 quads of No. 16 A. W. G. and 120 quads of No. 19 A. W. G. pure copper conductors in one of the principal sections, and the arrangement of the four wires in each quad is such that two physical circuits and one phantom circuit are made available. The method of obtaining three telephone circuits from two pairs of wires is old and extensively used. It is illustrated in Fig. 14. The method results in a 50 per cent increase in the number of available circuits and its application to this project is therefore of very great economic importance. Now the total of 139 quads multiplied by 3 gives 417 circuits or as many as could be carried on about 14 heavily loaded pole lines if aerial wire were used, but as will be described later, we will have to use two of these circuits to make one telephone circuit in some cases where the distances are comparatively great, so it is expected that only about 300 telephone circuits will be obtained for regular service. This is as many as could be carried on 10 heavily loaded pole lines if aerial wire were used. It is now thought that in some sections this number of circuits will take care of future demands for about 10 years after allowing for the dismantling of some existing aerial wire.

As these cables can be obtained in any size desired up to the maximum, the period for which they should be engineered can be determined from studies of circuit requirements and costs. These studies are of very great importance and the cost considerations include, of course,

annual costs of the various plans over proper periods as well as first costs.

Loading

Loading coils are now connected to many of the circuits and all of the circuits in this cable are intended to be equipped with coils located at 6000-foot intervals. The theory and practice of loading are described in papers previously presented before the Institute[1] and for our purpose it will be sufficient to state that these loading coils very materially reduce the attenuation losses and improve the quality of transmission as compared to cable circuits not so equipped. The improvement in so far as the attenuation losses are concerned, varies with the type of circuit and loading coils, but with one of the No. 19

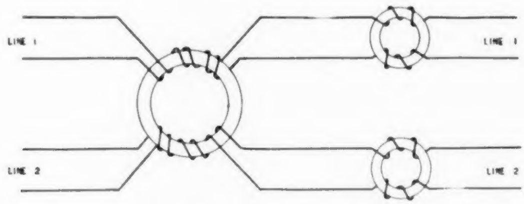


Fig. 15—Loading Coils Connected to a Group of Four Wires and Arranged for Phantom Operation

A. W. G. circuits in this cable loaded with coils having an inductance of 0.175 henry located at 6000-ft. intervals, the losses are only about one-third as great as in a similar circuit without the coils. The connections and arrangements of the coils are shown in Fig. 15 and it will be noted that coils have been connected to both the physical and phantom circuits. The arrangement is such that there is no appreciable interference between circuits due to magnetic action in the iron cores of the different coils or to the necessarily close electrical relation in the windings.

The loading coils are potted and sealed in iron pots, two of which are shown in Fig. 16, and in the country these are mounted on pole fixtures. Each pot contains 36 groups of 3 coils each. The pots are nearly 30 inches in diameter at the flange, 52 inches high and weigh about 2700 pounds. The pots can be obtained in different sizes depending upon the number of coils which it is desired to install at one time. When the cable was installed, extra lead sleeves were

[1] Papers by M. I. Pupin, Transactions of A. I. E. E., XVII, May 1900 and XV, March 1899.

Paper by Bancroft Gherardi, Transactions of A. I. E. E., XXX, June 1911.

placed at the loading points and a little slack left in the wire to facilitate the connection of four additional loading pots to the cable at

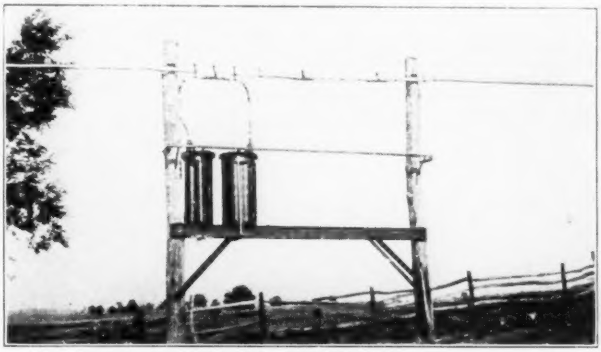


Fig. 16—Loading Fixture

some later date when the circuits are needed. The loading points must be uniformly spaced in order to obtain the proper impedance characteristics in the circuits as will be referred to later. Fig. 17 shows the iron core of a loading coil and Fig. 18 shows this core

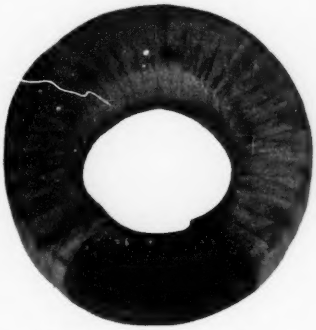


Fig. 17—Loading Coil Core

Fig. 18—Loading Coil with Winding Completed

wound with insulated wire and then wrapped with cloth and the terminals brought out nearly ready for potting. Fig. 19 shows several

coils arranged on one of the spindles which will be placed in the iron pot also shown. This particular pot will hold 7 spindles and when they are in place, the pot will be filled with compound and thoroughly sealed.

Telephone Repeaters

Even with the improvement in the quality of transmission and reduced attenuation losses effected by the use of loading coils, loaded

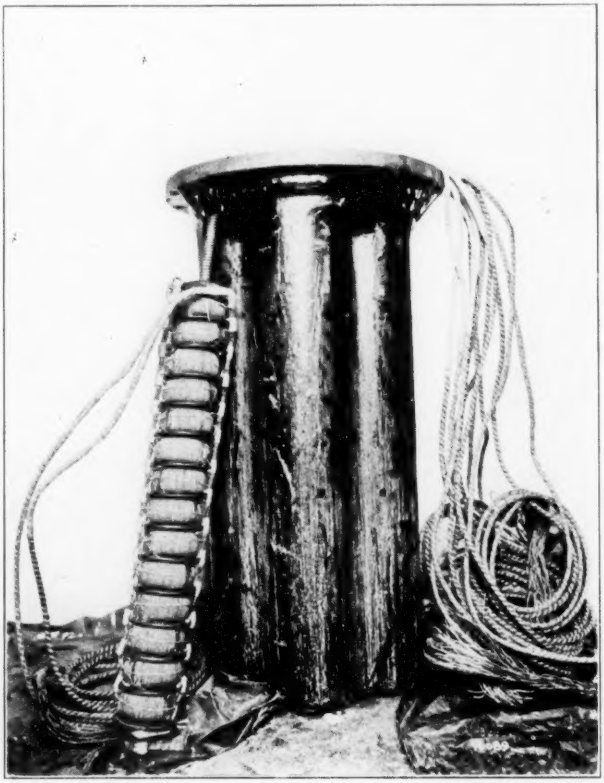


Fig. 19—Loading Coils on Spindle, Iron Loading Coil Case and Spindle Cables

cable circuits alone of No. 16 and No. 19 A. W. G. could be satisfactorily operated for distances less than 100 and 60 miles, respectively, and this is far short of our requirements in this case. In fact, we wish to operate some telephone circuits on these conductors and through this cable and future cables up to at least 1000 miles in length. This

can be accomplished by the use of telephone repeaters connected to the loaded conductors.

Telephone repeaters have been developed to a high state of perfection and are completely described in a paper presented by Messrs. Bancroft Gherardi and Frank B. Jewett at a joint meeting of the A. I. E. E. and the Institute of Radio Engineers in New York, October 1, 1919. Briefly, the purpose of a telephone repeater is to receive small telephone currents, amplify them and send them on, preserving

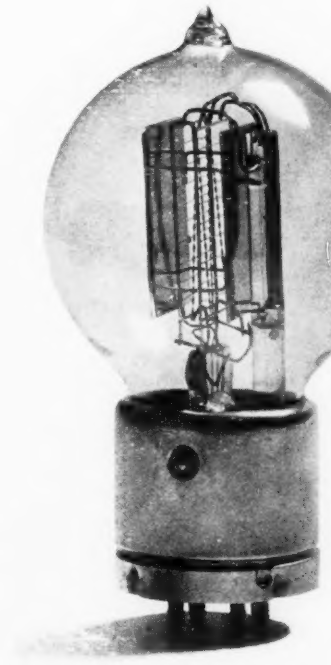


Fig. 20—Vacuum Tube

all the while the original wave shape. Therefore, if one or more telephone repeaters are properly inserted in circuits adapted to their use, the range of satisfactory transmission can be greatly extended. As many hundreds of vacuum-tube repeaters are in operation on the Philadelphia-Pittsburgh cable and connected cables, and as a great many more are planned for future installation, we will briefly consider the elementary features of some of the types of repeaters used.

Fig. 20 shows the structure of the vacuum tube which is an essential

element of this type of repeater. It is a small glass bulb with a vacuum that is as good as is practicable to obtain. In the tube is a filament which is heated to incandescence during operation,

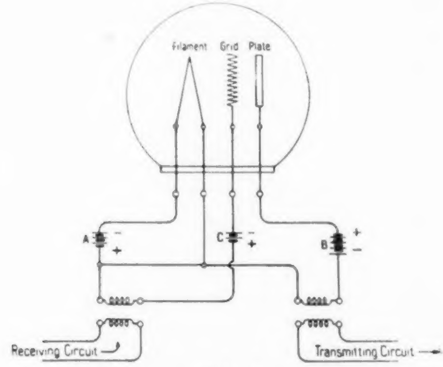


Fig. 21—Vacuum-Tube Repeater Element

and a grid and plate. The circuits directly associated with the tube are shown in more detail in Fig. 21, and this would constitute a device for amplifying currents from one direction. As is well understood, any change in the potential impressed on the grid causes a change in the current flowing in the plate-filament circuit. To obtain complete two-way repeater action two of these amplifier arrangements are combined with the circuits shown in Fig. 22.

Fig. 22—Two-Way Vacuum-Tube Repeater Circuit

It will be noted that the line circuit from one direction, for instance, the one designated "line west," is connected through a three-winding transformer to a balancing network which is so made up as to balance

the line as nearly as possible at telephone frequencies. This balance is essential to proper repeater operation. The circuit arrangement is such that part of the incoming energy is diverted to that part of the circuit containing the input coil directly associated with this three-winding transformer. By the action of the vacuum-tube arrangement amplified energy is transmitted to the line east. That part of the original incoming energy from the line west that goes through the balancing network or the output coil is not, of course, transmitted along into the line east. The operation in the case of currents incoming from the line east is similar and it will be noted that the complete repeater circuit is made up of two symmetrical parts. This circuit arrangement constitutes what is known as a two-wire repeater and the apparatus is, of course, all closely associated in the same office.

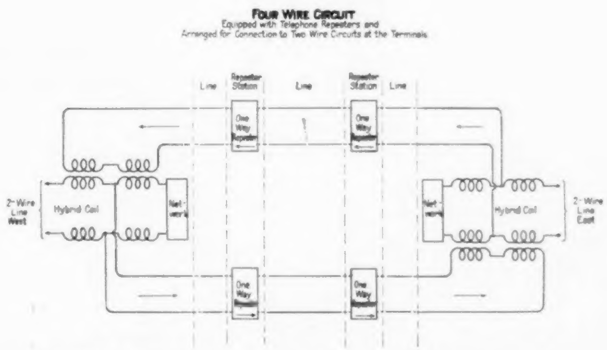


Fig. 23—Four-Wire Circuit equipped with telephone repeaters and arranged for connection to two wire-circuits at the terminals

Several of these repeaters may be inserted in tandem at appropriate points in a circuit, but there is a limit to the length of circuit that can be satisfactorily operated with this arrangement, this length depending upon the type of the facilities used. When longer circuits are required, a four-wire arrangement is used, as shown in Fig. 23. It will be noted that in this arrangement the three-winding transformers are not located in the same office but may be in offices several hundred miles apart. At each of the intermediate stations a vacuum-tube amplifier arranged for amplification in one direction only is connected to each of the two branches of the circuit. Two circuits are, of course, required between the terminals and these may be either physical or phantom circuits.

An advantage of this arrangement is that balancing networks are

not required at each repeater station and the general matter of balance and consequently good repeater operation in the circuit as a whole is greatly simplified. This arrangement can, therefore, be satisfactorily used for long circuits where two-wire operation might be impracticable,

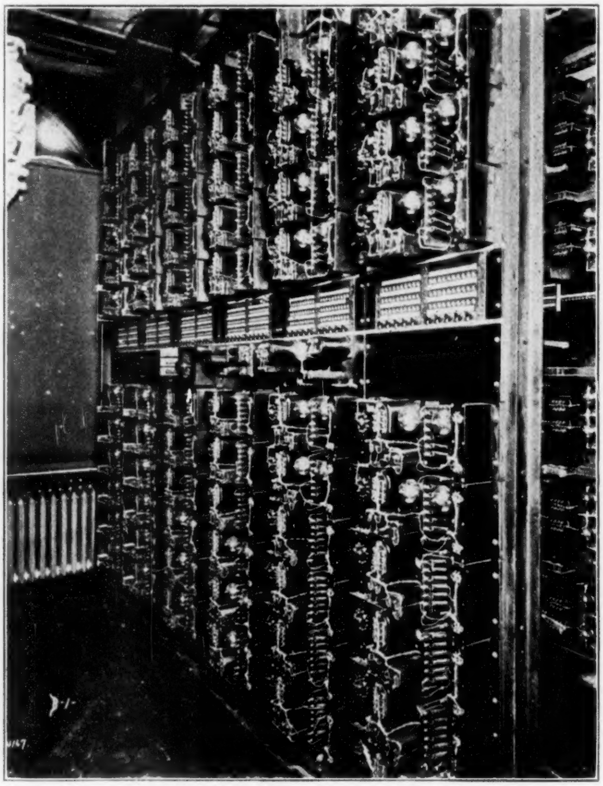


Fig. 24—Group of Repeaters at Reading, Pa.

and examples would be such circuits as New York-Pittsburgh or New York-Chicago.

Both of these types of circuits may be operated on No. 19 A. W. G. four-wire facilities which may be either physical or phantom circuits.

Fig. 24 shows a group of repeaters installed in the office at Reading, Pa., and Fig. 25 shows one of the four-wire repeater units in somewhat greater detail.

Line Impedance

In order that networks may be used to balance the lines for repeater operation, it is necessary as a practical proposition that the impedance characteristics of the lines be fairly uniform over the range of telephone frequencies. The solid line in Fig. 26 shows the resistance component of the impedance of a No. 19 loaded cable circuit with all loading coils in place. The solid line in Fig. 27 shows the

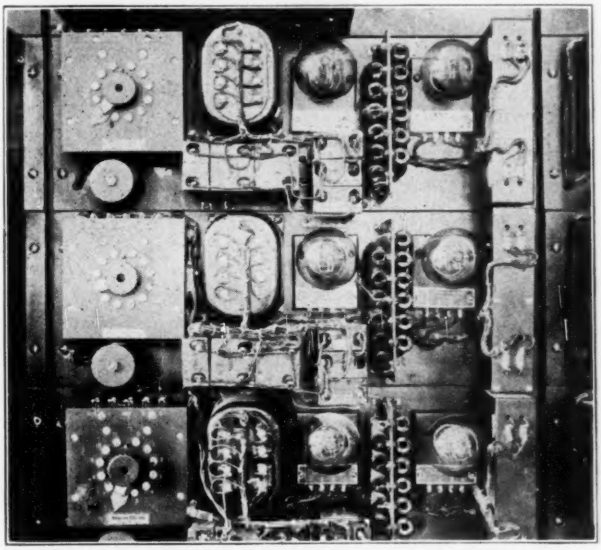


Fig. 25—Assembly of Four-Wire Repeater Apparatus

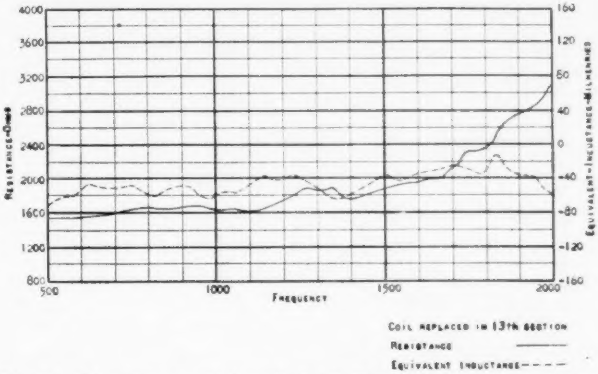


Fig. 26—Line Characteristics—A Cable Circuit in Normal Condition

resistance component found in impedance measurements on the same circuit with one coil omitted at the thirteenth loading point from the end at which the tests were made. It will be noted that in the latter case the characteristics of the circuits vary greatly with frequency. It would therefore be very difficult as a practical proposition to build up a network that would balance lines in this condition, and such variations in the electrical characteristics of a circuit impair the quality of telephone transmission, as the currents of different frequencies are differently affected. The necessity for careful maintenance work in promptly replacing loading coils which may become defective or preventing other irregularities from creeping into the plant will therefore be clear.

Transmission Regulation

The resistance of small-gage cable conductors is one of the important factors that determine the transmission losses of a circuit. The resistance of a No. 19 A. W. G. pair is about 88 ohms per mile so that

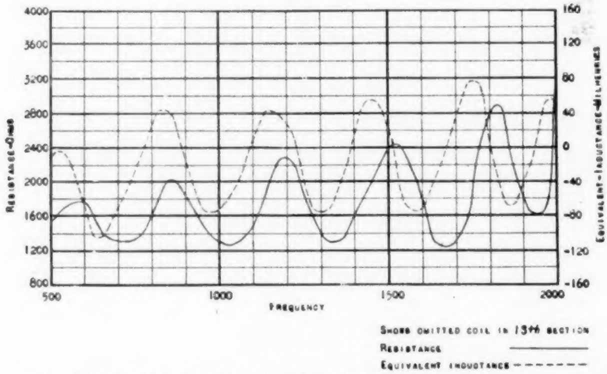


Fig. 27—Cable Circuit with Loading Coil Missing at Thirteenth Loading Point from Terminal

in a long circuit this factor of line resistance reaches considerable proportions. Now as most of the cable is aerial, the resistance of the conductors is of course affected by changes in temperature both daily and seasonal and the transmission losses vary accordingly. These changes in transmission values are of such magnitude that automatic transmission regulators are being provided for certain groups of longer circuits. All changes in the transmission equivalents of the circuits from whatever causes must be carefully watched and necessary adjustments made or the service will be seriously affected.

Telegraph

In the section between Philadelphia and Pittsburgh practically all of the existing long aerial wire circuits are composited, that is, they are arranged for simultaneous telephone and telegraph operation. The telegraph circuits thus obtained are generally used in furnishing what is sometimes called "leased wire" service. The ground return system providing either full duplex or single-line operation is used and the line currents average about 75 milliamperes. This grounded telegraph system cannot be used where simultaneous telephone and telegraph service is desired on loaded cable circuits of the length involved in this cable, and as a part of the work of carrying out the comprehensive toll cable plans of the Bell system, a new telegraph system had to be developed. It was found preferable to use a metallic return circuit and to limit the line current to a value between 3 and 5 milliamperes in order to prevent serious interference to the telephone circuits due to the "flutter effect,"[2] Morse thump, and mutual interference between telegraph circuits. Morse thump results when the composite sets, that is, the apparatus used for separating the telephone and telegraph currents, do not completely prevent the latter from entering the telephone circuit, thus causing interference. The telegraph repeaters are located at about 100-mile intervals on the No. 19 circuits and at somewhat less frequent intervals on No. 16 circuits. The telegraph apparatus is of course located in the same buildings that are used to house the telephone repeaters, and on the Philadelphia-Pittsburgh cable telegraph repeaters will be located initially at Philadelphia, Harrisburg, Bedford and Pittsburgh.

Test Boards

All of the conductors in the cables are carried into stations located at about 50-mile intervals and apparatus is provided in these stations for making regular tests to ascertain the condition of the cable and to locate trouble quickly. At these offices the different kinds of operating apparatus are also connected to the cable conductors; examples of this apparatus are phantom repeating coils, composite sets to permit simultaneous telephone and telegraph operation, telegraph repeaters, telephone repeaters and associated balancing equipment, signaling apparatus, and where required, the switchboards necessary for making the telephone connections involved in furnishing service. It is necessary that this apparatus which is installed in large quantities

[2]Paper by Martin and Fondiller in Journal of A. I. E. E., February, 1921.

be systematically arranged and facilities provided for making quick changes in the circuit arrangement. The circuits are wired through jacks installed in groups in test boards for this purpose and to facilitate testing. One of these boards is illustrated in Fig. 28. This particular board is located in one of the larger offices. The test boards in one of the repeater stations, such as Bedford, would consist of a smaller

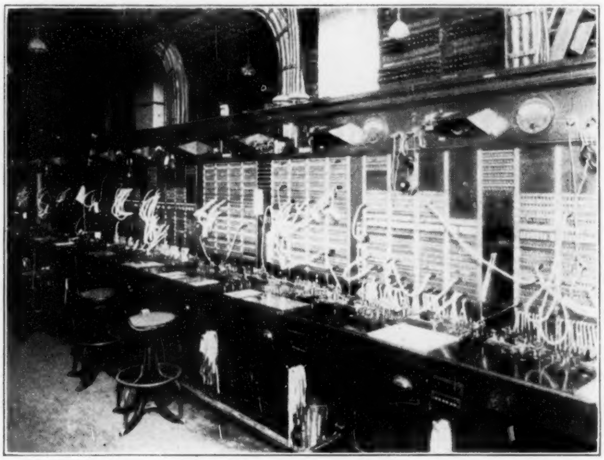


Fig. 28—Test Boards

number of positions. A position is three feet in length. In Fig. 28 each position bears a number.

Stations and Power Plants

Telephone repeaters of either the two-wire or four-wire type are connected to the circuits at approximate intervals of either 50 or 100 miles, depending upon the type of facilities which it is economical to use in the different circuits and the kind of service for which a given circuit is intended. As mentioned above, telegraph repeaters are installed at about 100-mile intervals. At some of these points existing offices are used while in a number of cases it was necessary to erect buildings for the sole purpose of housing the repeaters, testing apparatus and other equipment associated with the cable. For example, new buildings of fire-proof construction were erected at Shippensburg, Bedford and Ligonier. Fig. 29 is a view of the building at the latter point and the other two buildings are similar to this one, the dimensions being about 50 by 80 feet. Power plants are installed in these build-

ings to furnish current of the proper characteristics for operating the apparatus, and storage batteries are provided to insure uninterrupted service. As an indication of the size of these plants the 24-volt storage batteries installed for the initial load at Bedford have a capacity of 2240 ampere-hours and this provides about one day's reserve. The capacity can, of course, be increased as repeaters are added from time to time when additional circuits are needed. Storage batteries of smaller sizes supplying current at potentials of 30, 120 and 130 volts are also provided.

Examples of Circuit Arrangements

Fig. 30 shows two possible methods of building up a Philadelphia-Pittsburgh terminal circuit and Fig. 31, a method of building up a New York-Pittsburgh terminal circuit. In all three cases these telephone circuits are intended to have a transmission equivalent of about 12 miles of standard cable. Some Philadelphia-Pittsburgh

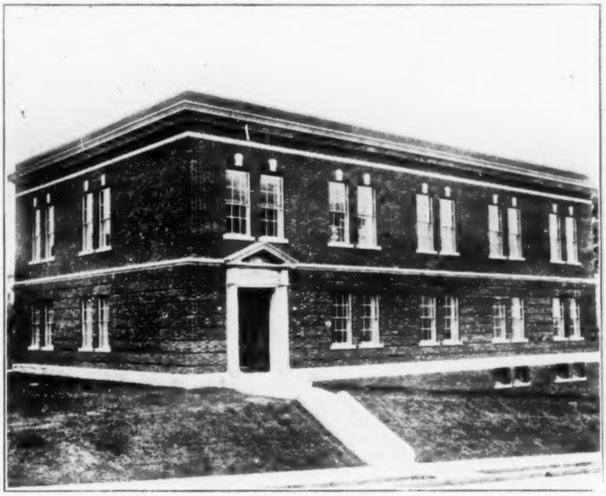


Fig. 29—Test and Repeater Station at Ligonier, Pa.

terminal circuits of the first type have been in everyday operation for several months, but it is not the most economical arrangement that it is possible to obtain for general use in providing this or similar service. It will be noted that No. 19 four-wire facilities are used between Philadelphia and Harrisburg and four-wire repeaters are located at these two points. At Harrisburg the four-wire circuit is

connected to a No. 16 two-wire circuit with a two-wire repeater at Bedford. This arrangement was used in order to start service through the cable with the facilities available, but it is intended later on to use the arrangement shown in example No. 2.

In example No. 2, No. 16 heavily loaded conductors are used and two-wire repeaters are located at Reading, Shippensburg and Ligonier. The total transmission equivalent of this circuit without repeaters is

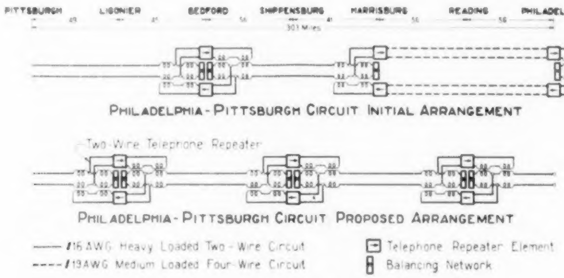


Fig. 30—Cable Circuit Arrangements

about 50 miles of standard cable so that in order to obtain a net equivalent of 12 miles for the circuit each of the three repeaters must give a transmission gain of nearly 13 miles of standard cable. This circuit could not of course be used for telephone purposes without repeaters.

The third example shows how it is expected to operate New York-Pittsburgh circuits intended for business between these two terminals.

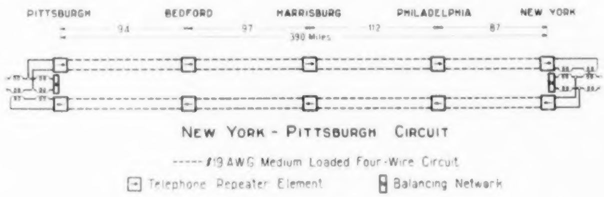


Fig. 31—Cable Circuit Arrangements

Four-wire No. 19 loaded cable facilities are used with four-wire telephone repeaters located at New York, Philadelphia, Harrisburg, Bedford and Pittsburgh.

Even with conductors of only two gages in the cable, it is clear that many different combinations of facilities can be built up into telephone circuits and an endeavor is always made to use the most eco-

nomical arrangement that will furnish the service required over each circuit group. The examples described above are of circuits used for business between the terminals indicated and if these circuits were to be connected to others extending to points considerable distances beyond these terminals different arrangements would be required. The cable conductors used in building up these telephone circuits can be composited and telegraph circuits are thus provided for simultaneous operation with the telephone circuits.

CONCLUSION

In the above discussion, an effort has been made to furnish some descriptive information regarding a complete cable system recently completed and now in successful operation between Philadelphia and Pittsburgh and designed for long-distance telephone and telegraph service. In one sense this discussion may be considered a report of the present status of the toll cable plant intended to connect Atlantic Seaboard cities with Chicago and other cities, and extensions are now under construction. However, most of the general methods which it is planned to use in these extensions are not expected to differ greatly from those described.

This cable system utilizes many new developments in the communication art and some of these, which have been briefly touched on here on account of their important application, have been described in more detail in previous papers. It is expected that more information regarding other specific developments which have contributed in an important way to the successful carrying out of this project or which may be utilized later on will be furnished in future papers.

An important feature of this cable project is the fact that while many new developments and practices are utilized, the design of the system as a whole is such as to fit in economically with existing wire and cable systems and proposed extensions.

Transmission Characteristics of the Submarine Cable[1]

By JOHN R. CARSON and J. J. GILBERT

SYNOPSIS: The present paper presents an extensive theoretical investigation of the impedance of the "sea return" of various types of submarine cables. In the case of the cables used for submarine telegraphy the impedance of the sea return has been practically negligible because of the low frequencies involved. For these low frequencies the cross-section of the return path is very large and its resistance low, even though the specific resistance of sea water is of the order of ten million times that of copper. As the frequency of the cable current is raised, however, the return currents crowd in nearer the cable and the resistance of the return path is increased. For frequencies in and above the telephone range, the return currents are forced into the steel armor wires around the cable and into the water just outside of the insulation. The small cross-section of the water involved and the loss in the armor wires cause the resistance of the return path to become a very large part of the total resistance of the circuit.

The present investigation led to the conclusion that the resistance of the return path could be greatly diminished by winding a low resistance conductor in the form of a copper tape immediately around the gutta percha insulation applied to the core of the cable. The concentric, cylindrical conductor thus formed lies within the armor wires but is not insulated from them and the sea water. Estimates of the sea return which would have been obtained in the Key West-Havana cable if no copper tape had been provided give values of 4, 6.5, and 8 ohms per nautical mile at 1,000, 3,000 and 5,000 cycles. The resistance actually obtained with the copper tape does not exceed 1.7 ohms at 5,000 cycles. The greater values would have increased the attenuation by approximately 30% at 1,000 cycles and by 50% at the two higher frequencies. The present cable permits of the operation of a carrier telegraph channel at 3,800 cycles, this lying above the range of telephone frequencies.

The paper gives a comparison of the theoretical conclusions with experimental data and the agreement is so satisfactory as to indicate that the theory is a reliable guide in the design of such a cable.—*Editor.*

I

THE transmission characteristics of a conducting system, such as a submarine cable circuit, are determined by its propagation constant, Γ, and characteristic impedance, K, which may be calculated for the frequency $p/2\pi$ from the formulas:

$$\Gamma = \sqrt{(R+ipL)(G+ipC)}, \tag{1}$$
$$K = \sqrt{\frac{R+ipL}{G+ipC}},$$

where R, L, G and C are the four fundamental line parameters, resistance, inductance, leakance, and capacity, all per unit length. These formulas are rigorous for all types of transmission systems; but the determination of the line parameters is not always possible by elementary methods, and may indeed be a matter of considerable com-

[1] Reprinted from the *Journal of the Franklin Institute*, December, 1921.

plexity and involve rather difficult analysis. In the case of the submarine cable, exact formulas are available for calculating the capacity and leakage and the core impedance. Considerable uncertainty is introduced into the theory, however, on account of the lack of a method of determining the "return impedance," that is, the contribution of the "sea return" (sea water, armor wires, etc.) to the effective resistance and inductance of the circuit. An investigation of this problem was undertaken by the writers in connection with the research program of the American Telephone and Telegraph Company and the Western Electric Company.

The purpose of the present paper is to discuss transmission over the submarine cable, and, more particularly, to develop rigorous formulas for the calculation of the impedance of the return conductor of the cable. The results of theoretical calculations are then compared with actual experimental data; and the agreement between theory and experiment is so satisfactory as to indicate that the former is a reliable guide in the design and predetermination of the cable.

Besides providing a method for accurately calculating the transmission characteristics of a submarine cable, the present analysis leads to the following general conclusions:

(1) Contrary to usual assumption, the "sea return" impedance is by no means negligible. Even at quite moderate frequencies there is a considerable crowding of the return current into the immediate neighborhood of the cable, with a consequent rapid increase of the resistance and a corresponding decrease of the inductance of the circuit. Except at the lowest frequencies, therefore, the impedance of the "sea return" is a very important factor.

(2) The armor wires which surround the cable, and which are necessary for mechanical protection, have a very pronounced effect on the impedance of the sea return, and even at moderate frequencies may become the controlling factor. Their action is to screen the current from the sea water itself, and, as the frequency increases, to carry more and more of the return current, until it is almost entirely confined to the armor wires and excluded from the sea water.

(3) The rapid increase in the impedance of the armor wires with frequency, and their pronounced and even controlling effect on transmission makes a thorough-going study of their role in the electrical system a matter of first-class importance. Heretofore they appear to have been regarded only as a mechanical protection, and their effect on transmission has been ignored. The accurate method of calculating their impedance which is developed in the following pages is believed to have considerable value in this connection.

(4) At relatively high frequencies, the return impedance, and hence the attenuation and the distortion, may be very greatly decreased by a correctly designed thin metallic sheath concentric with the core, and in electrical contact with the armor wires. The very important action of such a sheath, even when extremely thin, does not appear to have been adequately recognized or studied. It is suggested that the introduction of such a sheath affords a means of greatly increasing the range of frequencies which the cable can transmit.

The general problem of determining the transmission characteristics of a system consisting of an insulated conductor surrounded by a concentric ring of armor wires immersed in sea water is of considerable difficulty, since in this case the propagated wave must be represented as a set of component waves centered upon or diverging from the axes of the core and of the individual armor wires. The problem was first simplified by replacing the ring of armor wires by a cylindrical sheath, thus giving circular symmetry to the structure. The analysis of this case, however, showed that the effect of the iron sheath replacing the armor wires was so pronounced as to make this simplifying assumption of doubtful validity. The general problem was therefore attacked, and rigorous methods developed for calculating the effect of the armor wires upon transmission. The results in this case differ markedly from those obtained for the case of a continuous iron sheath, which indicates that great caution must be used in making assumptions regarding the physical structure of the armoring.

The present paper follows rather closely the course of the writers' investigation. In Section II is analyzed the problem of transmission over a system consisting of n coaxial cylindrical conductors, which may be either in electrical contact at their adjacent surfaces or separated from each other by dielectric spaces. The outermost conductor, consisting of the sea water, is assumed to extend to infinity. This analysis is then applied, in Section III, to the case of a submarine cable which is armored with a continuous iron sheath. This problem is not only of interest in itself, but serves as a first approximation to the case of an actual cable, and gives a clear qualitative idea of the effect of the various factors on transmission. In Section IV the problem of the submarine cable armored with a ring of iron wires is attacked and solved by rigorous methods, and the theoretical results are then compared with experimental data.

II

The solution of the problem of transmission of periodic currents over a system comprising n coaxial cylindrical conductors consists

in finding the particular solution of Maxwell's equations which satisfies the boundary conditions—continuity of tangential electric and magnetic forces at the surfaces of the conductors. Let the common axis of the conductors coincide with the Z axis of a system of polar coordinates, R, Φ, Z, and let the electric and magnetic variables involve the common factor $exp\ (-\Gamma z + ipt)$, Γ is therefore the propagation factor characterizing transmission, and p is 2π times the frequency. This factor will not be explicitly written in any of the work that follows, but it will be assumed to be incorporated in each of the electric variables so that

$$\frac{\partial^n}{\partial z^n} = (-\Gamma)^n, \qquad \frac{\partial^n}{\partial t^n} = (ip)^n.$$

From symmetry, it is evident that the component of electric field intensity in the direction of ϕ vanishes, and that the magnetic lines of force are circles lying in planes perpendicular to the axis of the system, and centered on that axis. Also, the axial and radial electric forces are independent of ϕ. It can be shown that the radial component of electric field intensity *in the conductors* is negligibly small compared with the axial component. The latter, for a given conductor, is of the form $E\ exp(-\Gamma z + ipt)$, where E is a solution of the differential equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + (\Gamma^2 - 4\pi\lambda\mu ip)E = O. \tag{2}$$

Here λ and μ are the electrical conductivity and the magnetic permeability of the particular conductor, measured in absolute electromagnetic units, and E is a function of r alone.

For the frequencies in which we are interested it may be shown that $\Gamma^2/4\pi\lambda\mu p$ is exceedingly small, so that (2) may be written

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} - 4\pi\lambda\mu ipE = O. \tag{3}$$

We will designate by the subscript j all quantities pertaining to the j^{th} conductor, counting from the axis. The solution of (3) for this conductor may then be written

$$E_j = A_j J_o(\rho_j) + B_j K_o(\rho_j), \tag{4}$$

where J_o and K_o are Bessel functions of zero order, A_j and B_j are arbitrary constants and

$$\rho_j = ri\sqrt{4\pi\lambda_j\mu_j pi} = r\alpha_j.$$

The magnetic field intensity can then be obtained from the curl law,

$$\mu \frac{dH}{dt} = \frac{dE}{dr},$$

which gives

$$H_j = \frac{\alpha_j}{\mu_j i p}\left[A_j J_o'(\rho_j) + B_j K_o'(\rho_j)\right], \tag{5}$$

where the prime indicates differentiation with respect to ρ_j. Taking the line integral of both sides of (5) around circular paths in conductor j lying close to the inner and outer surfaces of the cylinder we obtain

$$A_j J_o'(y_j) + B_j K_o'(y_j) = \frac{2\mu_j i p}{y_j}(I_1 + I_2 + \ldots + I_{j-1}), \tag{6}$$

$$A_j J_o'(x_j) + B_j K_o'(x_j) = \frac{2\mu_j i p}{x_j}(I_1 + I_2 + \ldots + I_j),$$

in which

I_j = current in the jth conductor,
$x_j = \alpha_j a_j$,
$y_j = \alpha_j b_j$,
a_j = external radius of jth conductor,
b_j = internal radius of jth conductor.

The values of the electric field intensity at the inner and outer surfaces of the j^{th} conductor can be written, from (4)

$$E_j' = A_j J_o(y_j) + B_j K_o(y_j),$$
$$E_j'' = A_j J_o(x_j) + B_j K_o(x_j).$$

Combining, in turn, each of these equations with relations (6) to eliminate A_j and B_j, we obtain

$$E_j' = Z_{j1}' I_1 + Z_{j2}' I_2 + \ldots + Z_{jj}' I_j \tag{7}$$
$$E_j'' = Z_{j1}'' I_1 + Z_{j2}'' I_2 + \ldots + Z_{jj}'' I_j,$$

in which

$$Z_{jk}'' = 2\mu_j i p\left[\frac{1}{x_j}\frac{J_o(x_j)K_o'(y_j) - J_o'(y_j)K_o(x_j)}{J_o'(x_j)K_o'(y_j) - J'(y_j)K_o'(x_j)} - \frac{1}{y_j}\frac{J_o(x_j)K_o'(x_j) - J_o'(x_j)K_o(x_j)}{J_o'(x_j)K_o'(y_j) - J_o'(y_j)K_o'(x_j)}\right], \quad k \neq j$$

$$Z_{jj}'' = \frac{2\mu_j i p}{x_j}\left[\frac{J_o(x_j)K_o'(y_j) - J_o'(y_j)K_o(x_j)}{J_o'(x_j)K_o'(y_j) - J_o'(y_j)K_o'(x_j)}\right],$$

$$Z_{jk}' = 2\mu_j i p\left[\frac{1}{x_j}\frac{J_o(y_j)K_o'(y_j) - J_o'(y_j)K_o(y_j)}{J_o'(x_j)K_o'(y_j) - J_o'(y_j)K_o'(x_j)} - \frac{1}{y_j}\frac{J_o(y_j)K_o'(x_j) - J_o'(x_j)K_o(y_j)}{J_o'(x_j)K_o'(y_j) - J_o'(y_j)K_o'(x_j)}\right], \quad k \neq j \tag{8}$$

$$Z_{jj}' = \frac{2\mu_j i p}{x_j}\left[\frac{J_o(y_j)K_o'(y_j) - J_o'(y_j)K_o(y_j)}{J_o'(x_j)K_o'(y_j) - J_o'(y_j)K_o'(x_j)}\right].$$

We have now succeeded in expressing the electric forces in the conductors as linear functions of the currents $I_1 \ldots I_n$, the coefficients being of the nature of impedances, by a method which is simply an application of the principle of continuity of magnetic field intensity. The remaining boundary condition, continuity of the tangential component of electrical field intensity gives, where two consecutive cylinders are in electrical contact,

$$E'_{j+1} - E''_j = \left[Z'_{j+1,1} - Z''_{j1} \right] I_1 + \ldots + \left[Z'_{j+1,j} - Z''_{jj} \right] I_j + Z'_{j+1,j+1} I_{j+1} = 0. \tag{9}$$

This gives m relations between the n currents of the system, m being the number of contacts between successive cylinders. In the case where the j and $(j+1)$st conductors are separated by a layer of dielectric material, a relation between the boundary values of electric field intensity may be obtained as follows:

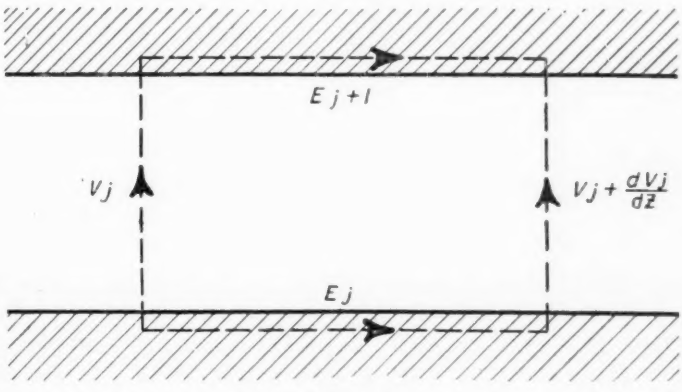
If E_r is the radial electric field intensity in the dielectric, then

$$V_{jj} = \int_{aj}^{bj+1} E_r dr$$

is the potential difference between the j and $(j+1)$st conductors, in the sense employed in ordinary circuit theory. If we now apply the law

$$\text{curl } E = -\mu \frac{dH}{dt}$$

$(j+1)^{st}$ Conductor



j^{th} Conductor

FIG. 1

to the elementary contour shown in Fig 1 we get

$$-\frac{\partial V_j}{\partial z} + E'_{j+1} - E''_j = \mu i p \Phi_j, \tag{10}$$

or

$$\Gamma V_j + E'_{j+1} - E''_j = \mu i p \Phi \tag{11}$$

where Φ_j is the magnetic flux threading the contour and is given by

$$\Phi_j = 2\,(I_1 + I_2 + \ldots + I_j) \log \frac{b_{j+1}}{a_j}.$$

From the law

$$\operatorname{div} kE = 4\pi Q,$$

$$E_r = \frac{2}{k_j r}(Q_1 + Q_2 + \ldots + Q_j)$$

where Q_j is the charge on the j^{th} conductor and k_j is the dielectric constant of the medium, whence,

$$V_j = \frac{2}{k_j} \log \frac{b_{j+1}}{a_j}\,(Q_1 + Q_2 \ldots + Q_j). \tag{12}$$

Furthermore, the rate of gain of charge is

$$\frac{\partial}{\partial t}(Q_1 + Q_2 + \ldots + Q_j) = -\frac{\partial}{\partial z}(I_1 + I_2 + \ldots + I_j) - 4\pi(Q_1 + Q_2 + \ldots + Q_j)\, g_j/k_j, \tag{13}$$

where the last term represents the leakage current, g_j being the specific conductivity of the dielectric.

From (13) we have

$$\frac{1}{k_j}(4\pi g_j + ipk_j)\,(Q_1 + Q_2 + \ldots + Q_j) = \Gamma\,(I_1 + I_2 + \ldots + I_j,)$$

and substituting this value of $(Q_1 + Q_2 + \ldots + Q_j)$ in (12) gives

$$V_j = 2\,(I_1 + I_2 + \ldots + I_j)\,\frac{\Gamma}{4\pi g_j + ipk_j} \log \frac{b_{j+1}}{a_j} \tag{14}$$

and from this and (11)

$$-\left[\frac{\Gamma^2}{G_j + ipC_j} - ipL_j\right](I_1 + I_2 + \ldots + I_j) = E'_{j+1} - E''_j \tag{15}$$

where

$$G_j = \frac{4\pi g_j}{2 \log \frac{b_{j+1}}{a_j}}, \quad C_j = \frac{k_j}{2 \log \frac{b_{j+1}}{a_j}}, \quad L_j = 2\mu_j \log \frac{b_{j+1}}{a_j}$$

Substituting the values of E_j'' and $F_j{}'_{+1}$ from (7) in (15) gives

$$- \left[\frac{\Gamma^2}{G_j + ipC_j} - ip\,L_j\right](I_1 + I_2 + \ldots + I_j) = (Z_j{}'_{+1\,1} - Z_{j,1}'')\,I_1 + \ldots \\ - Z_{j+1,j+1}' I_{j+1}. \tag{16}$$

An equation of this sort may be obtained for each layer of dielectric and these combined with equations (9) and the condition that the electric field intensity in the sea water must vanish at infinity,

$$E_n' = Z_{n1}''\,I_1 + \ldots + Z_{nn}''\,I_n = O,$$

give n relations between $I_1 \ldots I_n$. In order that these shall be consistent, the determinant of the coefficients must vanish.

$$\begin{vmatrix} Z_{21}' - Z_{11}'', & Z_{22}', & 0 & - - - 0 \\ Z_{31}' - Z_{21}'', & Z_{32}' - Z_{22}'', & Z_{33}', & - - - 0 \\ - & - & - & - - - - \\ Z_{j+1,1}' - Z_{j,1}'' + Z_j, & Z_{j+1,2}' - Z_{j,2}'' + Z_j, & - & - - - - \\ - & - & - & - - - - \\ Z_{n1}'', & Z_{n2}'', & - & - - - Z_{nn}'' \end{vmatrix} = O \tag{17}$$

where

$$Z_j = \frac{\Gamma^2}{G_j + ipC_j} - ipL_j.$$

This is an equation in Γ^2 of degree equal to the number of dielectric layers; consequently, there are as many independent modes of propagation in the system as there are branches in the network of conductors.

From this point the method of determining the behavior of the system depends upon conditions in the particular problem. For the case where there are k dielectric layers separating the conductors into $k + 1$ groups the current on the j^{th} group may be written in the form

$$I_j = A_{j1}\,exp\,(-\Gamma_1 z + ipt) + \ldots + A_{jk}\,exp\,(-\Gamma_k z + ipt) \\ + B_{j1}\,exp\,(\Gamma_1 z + ipt) + \ldots + B_{jk}\,exp\,(\Gamma_k z + ipt),$$

where $\Gamma^2{}_1 \ldots \Gamma^2{}_k$ are the k roots of the determinant (17) and $A_{j1} \ldots A_{jk}$, $B_{j1} \ldots B_{jk}$ are constants. These constants are not all in-

dependent, however, since, for each value of Γ, Γ_1 for instance, there exist k relations of the form (16) which the corresponding set of constants $A_{11}, A_{21}, \ldots A_{k1}$ must satisfy. The remaining $2k$ independent constants can then be determined from a knowledge of the conditions at the terminals of the conductors.

It is important to observe that the transmission characteristics of a system of coaxial conductors are influenced to a great extent by the manner of connecting the various members of the system. Anomalies in the impedance of a complicated network such as a submarine cable with several conducting sheaths in the return path, may often be traced to lack of proper connections between the sheaths, or to faulty joints.

III

The submarine cable armored with a continuous coaxial sheath, as shown in Fig. 2, is a particular case of the foregoing, and one which presents a clearer idea of the physical significance of the various steps in the general theory. There are only two groups of conductors, the

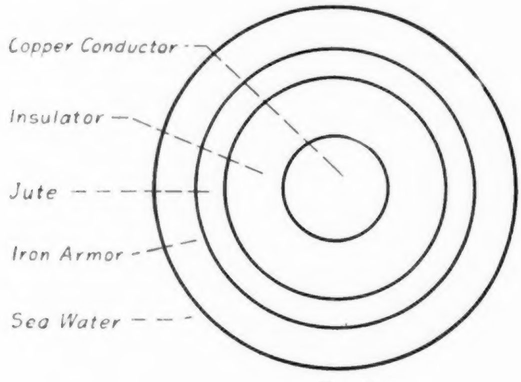


Fig. 2

first consisting of the core conductor, and the second comprising the iron sheath and the sea water, the two groups being separated by the insulating material and the layer of jute. Consequently, there is only one mode of propagation, and the analysis is considerably simplified.

The jute is assumed to contain sufficient sea water so that although it conducts practically no current axially, it maintains equality of potential between the outer surface of the gutta percha and the inner

surface of the iron sheath. Consequently equation (10) may be written

$$\frac{\partial V}{\partial z} - E_2' + E_1'' = - \mu i p \Phi = - i p L_{12} I_1, \tag{18}$$

where E''_1 and E'_2 are the values of electric field intensity at the outer surface of the core conductor and the inner surface of the iron, respectively, V is the potential difference between these two surfaces, and Φ is the magnetic flux threading unit length of the gutta percha and jute. Also, from (14)

$$- \frac{\partial V}{\partial z} = \frac{\Gamma^2}{G + ipC} I_1 \tag{19}$$

in which I_1 is the current in the core and

$$G = \frac{4\pi g_{12}}{2 \log \dfrac{b}{a_1}}, \qquad C = \frac{k_{12}}{2 \log \dfrac{b}{a_1}}, \tag{20}$$

where g_{12} and k_{12} are the electrical constants of the gutta percha, and b is the external radius of the core. It is evident, that G and C are respectively the leakage and capacity of unit length of the cable. Therefore, from (1),

$$\frac{\Gamma^2}{G + ipC} = R + ipL = Z, \tag{21}$$

where R and L are the resistance and inductance of unit length of the cable, including the sea return. Equation (18) may then be written

$$Z\, I_1 = E_1'' - E_2' + ipL_{12}\, I_1. \tag{22}$$

To determine Z we must express E''_1 and E'_2 as functions of I_1.

We have seen that

$$E_1'' = Z_1\, I_1, \tag{23}$$

where Z_1 may be termed the " internal impedance " per unit length of this conductor. In fact, when we place $y_1 = o$ in (8) we obtain

$$Z_{11}'' = \frac{2\mu_1 ip}{x_1} \frac{J_o(x_1)}{J_o'(x_1)}, \tag{24}$$

which is the usual formula for the internal impedance of a cylindrical conductor.

Similarly

$$E_2' = - Z_2\, I_1 \tag{25}$$

where Z_2 is the internal impedance of the return conductor, the

minus sign being due to the fact that the current in the return is in the negative direction of z.

Inserting (23) and (25) in (22) gives

$$Z = Z_1 + Z_2 + ipL_{12}.$$

The quantity Z_2 may be determined in the following manner. From (7) we have

$$E_2' = Z'_{21}\, I_1 + Z'_{22}\, I_2, \tag{26}$$

where I_2 is the current in the iron sheath. The value of this current can be found by applying the condition of continuity of electric field intensity at the common surface of the iron and the sea water, as in equation (9). This gives

$$Z''_{21}\, I_1 + Z''_{22}\, I_2 = Z'_{31}\, I_1 + Z'_{32}\, I_2 + Z_{33}\, I_3,$$

in which I_3 is the current in the sea water. From (8) it can be seen that $Z_{33} = O$, since $x_3 = \infty$, therefore

$$I_2 = \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}}\, I_1. \tag{27}$$

Substituting (27) in (26) gives

$$E_2' = \left[Z'_{21} + \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}}\, Z'_{22} \right] I_1,$$

and by comparison with (25) we have

$$Z_2 = -\, Z'_{21} - \frac{Z'_{31} - Z''_{21}}{Z''_{22} - Z'_{32}}\, Z'_{22} \tag{28}$$

as the internal impedance of the return conductor. The resistance and reactance per unit length of this portion of the circuit are then represented by the real and imaginary parts of (28) respectively.

We may then determine R and L from the formula

$$Z = R + ipL = Z_1 + Z_2 + ipL_{12}, \tag{29}$$

where Z_1 and Z_2 are calculated from (23) and (28) and

$$L_{12} = 2 \log \frac{b_2}{a_1},$$

b_2 and a_1 being the inner radius of the iron and the outer radius of the core conductor, respectively.

For purposes of comparison, the return impedance is calculated for the case where the iron armoring is absent, the return current

being conducted by the sea water alone. As in the preceding case,

$$Z_1 = \frac{2\mu_1 ip}{x_1} \frac{J_o(x_1)}{J_o'(x_1)}$$

The expression for Z_2 simplifies considerably. The electric field intensity in the sea water may be written, from (4),

$$E_2 = B_2 K_o(\rho_2), \tag{30}$$

the term in J_o being absent in order to permit E_2 to vanish at infinity.

Also, from (6),

$$B_2 K_o'(y_2) = \frac{2\,\mu_2 ip}{y_2} I_1. \tag{31}$$

From (30) and (31) we have

$$E_2' = \frac{2\mu_2 ip}{y_2} \frac{K_o(y_2)}{K_o'(y_2)} I_1. \tag{32}$$

from which the return impedance can be written,

$$Z_2 = -\frac{2\mu_2 ip}{y_2} \frac{K_o(y_2)}{K_o'(y_2)} \tag{33}$$

We have then

$$Z = R + ipL = \frac{2\mu_1 ip}{x_1} \frac{J_o(x_1)}{J_o'(x_1)} - \frac{2\mu_2 ip}{y_2} \frac{K_o(y_2)}{K_o'(y_2)} + ipL_{12}.$$

The resistance and inductance of the sea return of a submarine cable were calculated from formula (28), employing the following values for the constants:

Copper $\begin{cases} a_1 = .226 \text{ cm.} \\ b_1 = 0 \\ \mu_1 = 1 \\ \lambda_1 = 6.06 \times 10^{-4} \end{cases}$

Iron $\begin{cases} a_2 = .990 \text{ cm.} \\ b_2 = .737 \text{ cm.} \\ \mu_2 = 100 \\ \lambda_2 = 8 \times 10^{-5} \end{cases}$

Sea Water $\begin{cases} a_3 = \infty \\ b_3 = .990 \text{ cm.} \\ \mu_3 = 1 \\ \lambda_3 = 5 \times 10^{-11} \end{cases}$

The armoring was then assumed to be replaced by sea water, and the resistance and inductance of the cable were calculated from (33).

The results of the calculations are shown in the curves of Fig. 3.

It is evident from these curves that the effect of the iron armoring is to increase considerably the impedance of the return path. The

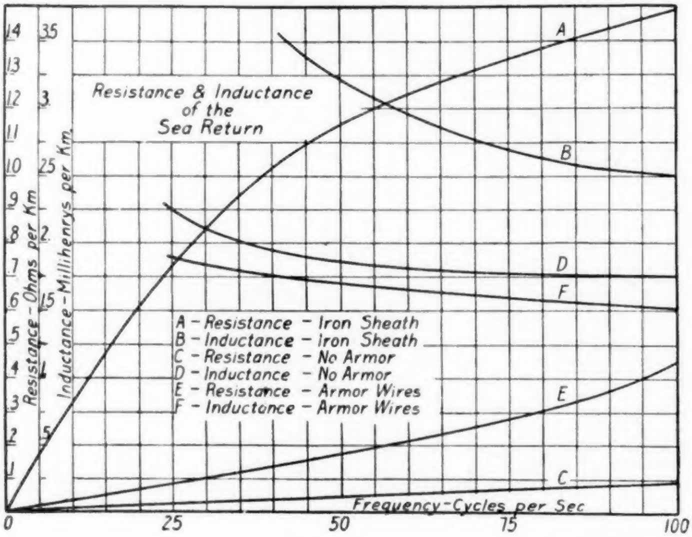


Fig. 3

physical explanation of this fact is that the iron acts as a shield to screen from the sea water the electromagnetic effects of the current flowing in the cable conductor. Energy is dissipated in the armoring and is prevented from spreading out through the surrounding medium. The assumption that the armor wires could be replaced by a solid cylinder of iron is, therefore, subject to question, since it is possible that the larger surface area of the assemblage of armor wires, and the gaps between these wires may be effective in diminishing the energy dissipated in the armoring and consequently diminishing the screening effect. This problem is investigated in the following section.

IV

The physical system under consideration is shown schematically in cross-section in Fig. 4, and consists of an insulated conductor and

protective covering of jute, surrounded by a ring of N armor wires immersed in sea water. The method of solution is essentially similar to that given in the preceding pages, and consists in determining the values of electric field intensity at the outer surface of the core conductor and the inner surface of the return conductor, from which the internal impedances of the two conductors can be found.

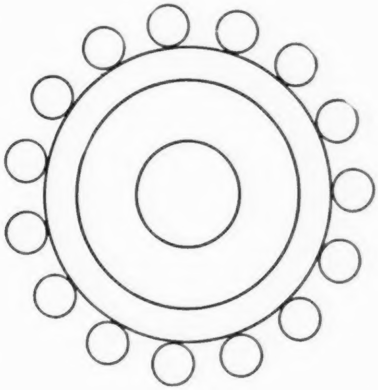


FIG. 4

The main difficulty in the analysis is caused by the lack of uniaxial symmetry in the return conductor. This was overcome by employing a method developed by one of the authors[2] in a study of transmission in parallel wires.

The electric field intensity in the sea water satisfies the differential equation

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E}{\partial \phi^2} - 4\pi\lambda\mu p i E = O,$$

the solution of which is a Fourier-Bessel expansion,

$$E = A_o K_o(r\alpha) + A_1 K_1(r\alpha)\cos\phi + A_2 K_2(r\alpha)\cos 2\phi + \ldots +,$$

r and ϕ being referred to the axis of the particular wire.

Assuming that the current distribution in the core conductor is independent of the angle ϕ, that is, neglecting the individual character of the armor wires only in their effect on the current distribution in the core, the effect due to the current in the core is represented

[2] "Wave Propagation over Parallel Wires: The Proximity Effect." John R. Carson, *Phil. Mag.*, vol. xli, p. 607 (1921).

by the first term of such a series, and the total field intensity may be written

$$E = A\,K_o\,(r\alpha) + \sum_{j=o}^{N-1} \sum_{s=o}^{\infty} B_s K_s\,(\alpha\rho_j)\cos s\phi_j, \tag{34}$$

ρ_j and ϕ_j being referred to the axis of wire j, as shown in Fig. 5. That is, the resultant field is expressible as a set of waves centered on the axis of the cable and the axes of the N armor wires.

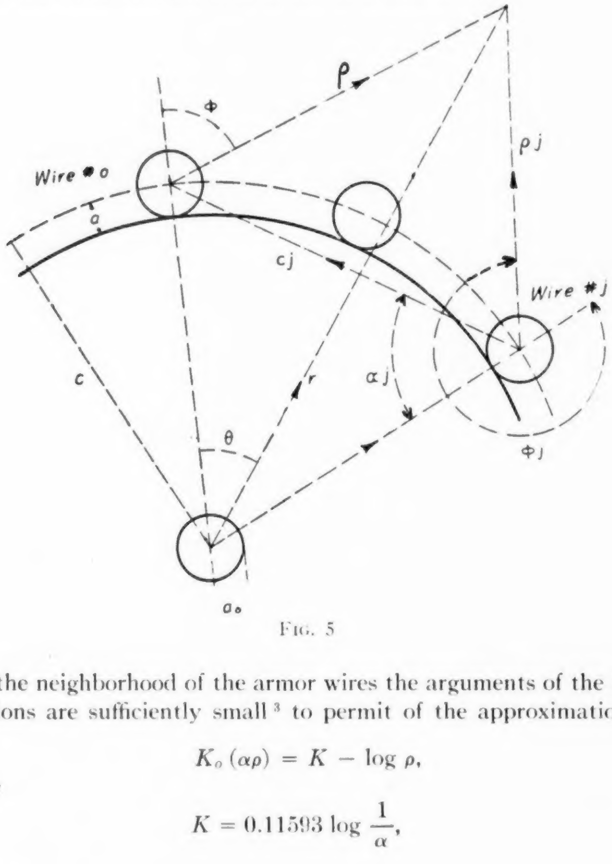


FIG. 5

In the neighborhood of the armor wires the arguments of the Bessel functions are sufficiently small [3] to permit of the approximations

$$K_o\,(\alpha\rho) = K - \log\rho,$$

where

$$K = 0.11593 \log\frac{1}{\alpha},$$

and

$$K_s\,(\alpha\rho) = \frac{1}{(-\alpha\rho)^s}.$$

[3] See Note I at end of paper.

The series (34) can, therefore, be written

$$E = A(K - \log r) + B_o(NK - \sum_{j=o}^{N-1} \log \rho_j) + \sum_{j=o}^{N-1} \sum_{s=1}^{\infty} B_s \frac{\cos (s\phi_j)}{\rho_j^s}, \quad (35)$$

in which B_s has absorbed the constant quantities. From this, the magnetic intensity in the sea water can be obtained by differentiation.

Inside any armor wire, at the surface, the field intensities are

$$E = C_o J_o(\xi) + C_1 J_1(\xi) \cos \phi + \ldots + C_n J_n(\xi) \cos n\phi + \ldots +, \quad (36)$$

$$H_\Phi = \frac{1}{a\mu i p}\left[C_o J_o'(\xi) + C_1 J_1'(\xi)\cos\phi + \ldots + C_n J_n'(\xi)\cos n\phi + \ldots + \right] (37)$$

where $\xi = ai\sqrt{4\pi\lambda\mu p i}$,

λ and μ being the electrical conductivity and the magnetic permeability, respectively, of the material of the armor wire. The quantities a and ϕ are centered on the axis of the wire.

In order to determine the coefficients A, B_o, B_1, $-$, C_o, C_1, $-$ we make use of the fact that the electric and the magnetic field intensities are continuous at the surface of the wire. It is obvious, however, that nothing can be learned by equating (35) and (36) since they are formally dissimilar. We therefore transform [4] the various terms of (35) to a common axis which coincides with the axis of one of the armor wires, hereafter called wire "zero," and the electric field intensity in the sea water, close to the surface of the armor wire, is

$$\begin{aligned} E = {} & (A + NB_o)K - A \log c - B_o \log (ac_1c_2 \ldots c_{n-1}) - \Sigma_o \\ & + \left[q_1/\zeta - \zeta (A + S_{11} B_o) + \frac{\zeta}{1'!} \Sigma_1 \right] \cos \phi \\ & + \left[q_2/\zeta^2 + \frac{\zeta^2}{2} (A + S_{22} B_o) + \frac{\zeta^2}{2'!} \Sigma_2 \right] \cos 2\phi \qquad (38) \\ & - - - - - - - - - \\ & + \left[q_n/\zeta^n + \frac{(-\zeta)^n}{n} (A + S_{nn} B_o) - \frac{(-\zeta)^n}{n'!} \Sigma_n \right] \cos n\phi, \end{aligned}$$

where

$$\begin{aligned} \Sigma_0 &= S_{11}q_1 - S_{22}q_2 + S_{33}q_3 \ldots, \\ \Sigma_1 &= S_{02}q_1 - 2\, S_{13}q_2 + 3\, S_{24}q_3 \ldots, \qquad (39) \\ \Sigma_2 &= 1.2\, S_{13}q_1 - 2.3\, S_{04}q_2 + 3.4\, S_{15}q_3 \ldots, \\ \Sigma_3 &= 1.2.3\, S_{24}q_1 - 2.3.4\, S_{15}q_2 + 3.4.5\, S_{06}q_3 \ldots, \\ & - - - - - - - - - \end{aligned}$$

[4] See Note II.

$$
\begin{aligned}
S_{pq} &= \sum_{j=1}^{N-1} \frac{\cos pa_j}{\left(2\sin\frac{j\pi}{n}\right)^q}, \\
q_n &= B_n/c^n, \\
\zeta &= \frac{a}{c}.
\end{aligned} \tag{40}
$$

The quantities ϕ and a have the same significance as in equations (36) and (37).

The tangential magnetic field intensity in the sea water at the surface of wire " zero " is, therefore,

$$
H_\Phi = \frac{1}{ip}\frac{dE_z}{da} = \frac{1}{ipa}\Bigg[-B_o + \cos\phi\left[-q_1/\zeta - \zeta(A + S_{11}B_o) + \frac{\zeta}{1!}\Sigma_1 \right] \tag{41}
$$

$$
+ 2\cos 2\phi\left[-q_2/\zeta^2 + \frac{\zeta^2}{2}(A + S_{22}B_0) - \frac{\zeta^2}{2!}\Sigma_2 \right]
$$

$$
- - - - - - - - - - -
$$

$$
+ n\cos n\phi\left[-q_n/\zeta^n + \frac{(-\zeta)^n}{n}(A + S_{nn}B_o) - \frac{(-\zeta)^n}{n!}\Sigma_n \right] - - - \Bigg]
$$

To satisfy the condition of continuity of electric and magnetic field intensities at the surface of the armor wire it is necessary that the coefficients of the corresponding terms of (36) and (38) and of (37) and (41) be equal. This gives

$$
C_o J_o(\xi) = (A + NB_o)K - A\log c - B_o \log(ac_1 \ldots c_n) - \Sigma_o, \tag{42}
$$

$$
C_n J_n(\xi) = q_n/\zeta^n + \frac{(-\zeta)^n}{n}(A + S_{nn}B_o) - \frac{(-\zeta)^n}{n!}\Sigma_n, \; n = 1, 2, \ldots \infty \tag{43}
$$

$$
\xi J'_o(\xi)\, C_o = -\mu B_o, \tag{44}
$$

$$
\xi J_n'(\xi)\, C_n = -n\mu\left[q_n/\zeta^n - \frac{(-\zeta)^n}{n}(A + S_{nn}B_o) - \frac{(-\zeta)^n}{n!}\Sigma_n \right].\; n = 1, 2, \ldots \infty \tag{45}
$$

From these expressions the quantities $B_1 \ldots C_1 \ldots$ can be determined. Multiplying (43) by $n\mu$ and subtracting (45) gives

$$
C_n = \frac{2n\mu q_n}{\zeta^n}\frac{1}{n\mu J_n(\xi) - \xi J_n'(\xi)}, \tag{46}
$$

which expresses C_n in terms of q_n. Multiplying (43) by $\zeta J_n'(\zeta)$ and (45) by $J_n(\zeta)$, and subtracting gives

$$q_n = (-1)^n \lambda_n \zeta^{2n} \left[\frac{1}{n}(A + S_{nn}B_o) - \frac{1}{n!}\Sigma_n \right], \; n = 1, 2, \ldots \infty \quad (47)$$

where

$$\lambda_n = \frac{n\mu J_n(\xi) - \xi J_n'(\xi)}{n\mu J_n(\xi) + \xi J_n'(\xi)}. \quad (48)$$

From the infinite set of simultaneous equations (47) the infinitely many variables q_n may be determined in terms of A and B_o.[5]

We have thus determined the arbitrary constants $C_o \ldots C_n$ and $q_1 \ldots q_n$ (or $B_1 \ldots B_n$) as functions of A and B_o. It remains to express the latter quantities in terms of physical quantities. If I_1 is the current in the armor then $\frac{I_1}{N}$ is the current in a single wire. Integrating (41) completely around the armor wire "zero" gives, therefore,

$$2pi\frac{I_1}{N} = -B_o. \quad (49)$$

Similarly, if I_o is the current in the core conductor, we find

$$2piI_o = -A. \quad (50)$$

We can, therefore, express all the arbitrary constants as linear, homogeneous functions of I_o and I_1.

To determine the relation between these currents, we have from (49) and (44),

$$CJ_o(\xi) = \frac{ZI_1}{N}, \quad (51)$$

where

$$Z = \frac{2\mu ip}{\xi}\frac{J_o(\xi)}{J_o'(\xi)}.$$

Substituting (49), (50) and (51) in (42) gives

$$\frac{Z}{N}I_1 = -2ip(I_o + I_1)K + 2ipI_o \log c + 2ip\frac{I_1}{N}\log(ac_1 \ldots c_n) \quad (52)$$
$$-(S_{11}q_1 - S_{22}q_2 + S_{33}q_3 - \ldots),$$

from which, since $q_1 \ldots q_n$ are functions of I_1 and I_o, the ratio I_o/I_1 can be obtained.

[5] See Note III.

Having shown that the constants A, B_o . . . of the series (35) are proportional to I_o, we can express the electric field intensity at the inner surface of the return conductor in the form

$$E_2 = -Z_2 I_o.$$

The computation of Z_2 is facilitated by transforming the terms of (35) to the axis of the core conductor [6] and placing $r = c - a$. We thus obtain

$$E_2 = -Z_2 I_o = (A + NB_o)K - A\log(c-a) - NB_o \log c - N(q_1 - q_2 + q_3 \ldots) \\ + (\text{terms containing } \cos\theta, \cos 2\theta, \text{etc., as factors}). \quad (53)$$

We have, by applying the curl law to an elementary contour which links the core conductor and the return,

$$\frac{\partial V}{\partial z} - E_1 + E_2 = -ip\Phi_{12}, \quad (54)$$

where

$$E_1 = Z_1 I_o = \frac{2\mu_o ip}{\xi_0} \frac{J_o(\xi_o)}{J_o'(\xi_o)} I_o,$$
$$\Phi_{12} = L_{12} I_o = 2 I_o \log \frac{c-a}{a_o}, \quad (55)$$

and

$$\xi_o = a_o i \sqrt{4\pi\lambda_o\mu_o ip},$$

λ_o and μ_o being the electrical constants of the core conductor and a_o its radius. The value given above for Φ_{12} holds only for the contour on which E_2 is independent of the angle θ, that is, when the terms of (53) that contain $\cos\theta$, $\cos 2\theta$, etc., vanish. The value of Z_2 to be used in (54) is therefore determined from

$$E_2 = -Z_2 I_o = (A + NB_o) K - A \log (c - a) - NB_o \log c \\ - N (q_1 - q_2 + \ldots) \quad (56)$$

As before,

$$-\frac{\partial V}{\partial z} = (R + ipL) I_o, \quad (57)$$

where R and L are the resistance and inductance per unit length of the cable, including the sea return.

We have then from (54),

$$R + ipL = Z_1 + Z_2 + ipL_{12}, \quad (58)$$

from which R and L can be determined.

[6] See Note II.

The process of calculating the resistance and inductance of a submarine cable by the method just described may be summarized as follows:

(1) Determine from (47) the quantities $q_1 \ldots q_n$ in terms of A and B_o, and then in terms of I_1 and I_o by (49) and (50).

(2) Substitute these values of $q_1 \ldots q_n$ in (52) and obtain the ratio I_o/I_1.

(3) Substitute for A, B_o and $q_1 \ldots q_n$ in (56) their values in terms of I_o and I_1.

(4) Eliminate I_1 from these two relations, thus obtaining E_2 in terms of I_o. Then $Z_2 = -E_2/I_o$.

(5) Substitute this value of Z_2 and the value of Z_1 calculated from (55) in equation (58).

(6) The resistance and the inductance per unit length of the cable may then be determined from the real and imaginary parts of the latter equation.

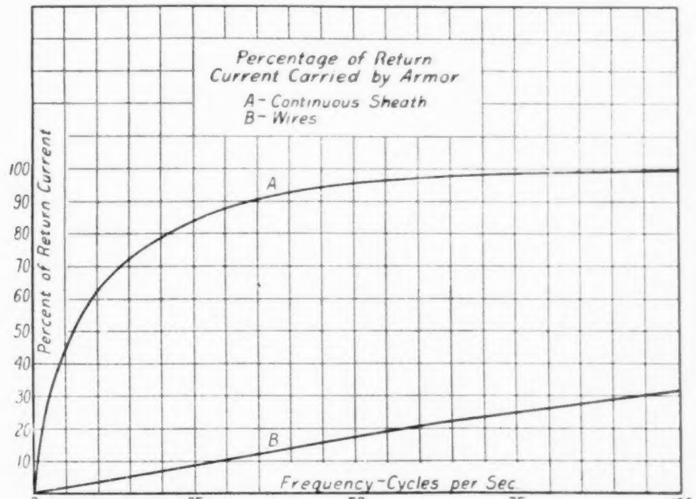


FIG. 6

The resistance and inductance of a cable of cross-section shown in Fig. 4 were computed by the method just described, the results being given by curves E and F of Fig. 3. The cable in this case is identical with that shown in Fig. 2 previously described, except that the continuous iron sheath has been replaced by fifteen wires. The

effect of the presence of the iron upon the resistance of the return conductor is still noticeable, although it is much less than in the case of the continuous iron sheath. The reason for this is evident after inspection of the curves of Fig. 6, which show the percentage of return current carried by the armor in the two cases. Especially at the lower frequencies, the return current is much more confined by the continuous sheath than it is by the wires.

As a check of the method, the resistance and inductance of the Seattle-Sitka cable of the United States Signal Corps were calculated for frequencies in the range 50 to 600 cycles per second, and

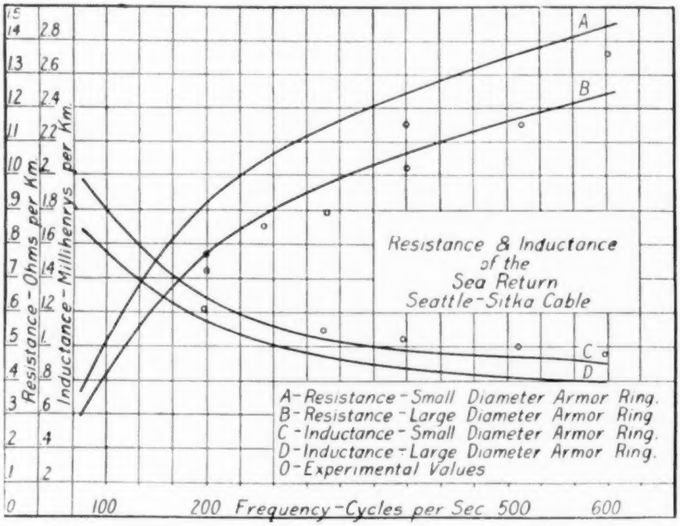


FIG. 7

the values so obtained were then compared with the results of measurements recently made upon this cable.[7] The constants used in the calculations were as follows:

Conductor
- Diameter .... .216 cm.
- Resistance per nautical mile .... 9 ohms

Rubber Insulation
- Outside diameter .... .718 cm.
- Capacity per nautical mile .... .38 mf.

Armoring
- 16 wires .... each .242 cm. diameter

Outside Diameter of Cable .... 2.06 cm.

[7] "The Use of Alternating Currents for Submarine Cable Transmission," Frederick E. Pernot, *Jour. of the Franklin Institute*, vol. 190, p. 323, 1920.

Owing to lack of information concerning the mean radius of the ring of armor wires, two sets of data were computed employing the values $c = 0.6148$ and $c = 0.920$, which correspond, respectively, to zero and maximum separation of the armor wires.

The results of the calculations are shown in Fig. 7. The experimental values are indicated by small circles, and agree well with the theoretical values throughout the range of frequencies. The re-

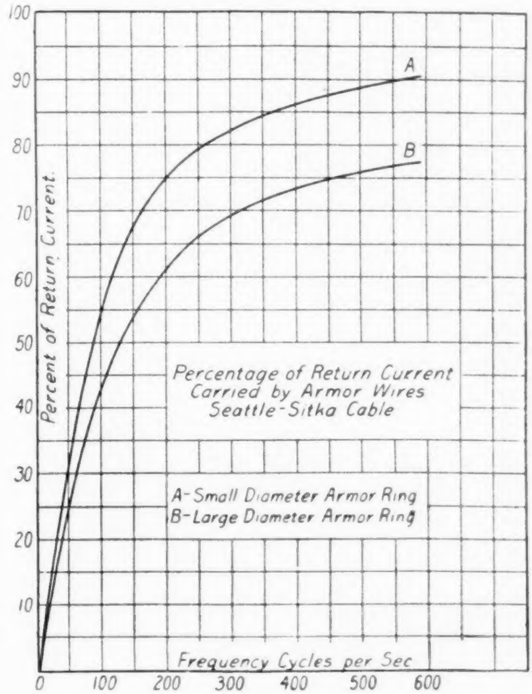


FIG. 8

sistance of the sea return increases most rapidly in the region of frequencies used in ordinary telegraphy, 0 to 100 cycles per second. In this range the inductance of the cable also has its greatest values, and these two effects have considerable influence in determining the transmission characteristics of the cable.

The percentage of the return current that is carried by the armor wires is shown in Fig. 8.

Conclusions

As was previously pointed out, the effect of the shielding action of the iron armor of a submarine cable is to diminish the electromagnetic field which is propagated through the sea water, and which gives rise to the return current. Combined with this effect is the shielding action of the sea water adjacent to the cable, upon the distant portions. The total shielding effect increases with the frequency until a point is reached where practically the whole of the return current is carried by the armor wires.

Several remedies have been suggested for diminishing the damping effect of the armor wires. It can be proved, for example, that for a given size of core and weight of armor, the number and size of armor wires can be chosen so as to give a minimum value of return impedance. A proper choice of the electrical constants of the material of which the armor is constructed would also be of advantage, since the return impedance is somewhat larger for iron than it is for material of higher or lower conductivity.

Another method of diminishing the return impedance, which has been used in practice, is to wrap the cable core with a number of concentric layers of conducting tape before it is covered with jute. The return current, as it crowds in toward the core with increasing frequency, will then have a path of comparatively low impedance, and at the higher frequencies only a small portion of the current will be carried by the armor wires and the sea water. The impedance of the return path can be calculated for this case by the methods given in the preceding pages. The following table compares the values of the resistance of the return conductor calculated by three different methods, and determined experimentally, for a cable provided with a brass tape 5 mils in thickness.

Resistance of Return Conductor—OHMS per Statute Mile

Frequency Cycles per Sec.	Approximate Method	Approx. Method[8] Corrected by Factor $\frac{2}{\pi}$	Exact Method	Experimental
3,000	4.00	3.15	2.87	2.92
10,000	4.90	4.25	4.45	4.60

The experimental values are the results of a series of measurements made by the Department of Development and Research of

[8] This is an empirical formula which has been found to be fairly close in most cases. The correction factor suggested itself in that it takes care of the increased surface of the armor wires, as compared with the corresponding continuous sheath.

the American Telephone and Telegraph Company upon the Victoria-Vancouver submarine cable. The calculated values were obtained by both the approximate and the exact methods, discussed in the preceding pages, in which the armor of the cable is treated, respectively, as a continuous sheath and as a ring of wires. The modifications which must be introduced to include the effect of the conducting tape are outlined in the discussion of the general theory. The agreement between the calculated and the measured values of return resistance proves that the method developed in the present paper is accurate even at the highest frequencies employed in telephony.

Note I—Note on Bessel Functions

The Bessel Functions of zero order of the first and second kinds, $J_o(\rho)$ and $K_o(\rho)$, used in the preceding work are all to a complex argument $\rho = iq\sqrt{i}$ where q is a real number and $i = \sqrt{-1}$. The following formulas[9] may be used for determining the values of these functions:

$q < 0.1$

$$J_o(\rho) = 1 \qquad\qquad J'_o(\rho) = -\tfrac{1}{2}\rho$$

$$K_o(\rho) = \log_e \frac{2}{\gamma\rho} = .11593 - \log_e q - \frac{\pi i}{4}$$

$$K_o'(\rho) = -\frac{1}{\rho}$$

(*Jahnke u. Emde*, "Funktionentafeln," pp. 97, 98.)

$0.1 < q < 10$

The reports of the British Association for 1912 and 1915 give the values in this range of the functions ber q, ber′ q, bei q, bei′ q, ker q, ker′q, kei q, kei′q which are defined by the relations

$$\begin{aligned} J_o(iq\sqrt{i}) &= \operatorname{ber} q + i \operatorname{bei} q, \\ i\sqrt{i}\, J_o'(iq\sqrt{i}) &= \operatorname{ber}' q + i \operatorname{bei}' q, \\ K_o(iq\sqrt{i}) &= \operatorname{ker} q + i \operatorname{kei} q, \\ i\sqrt{i}\, K_o'(iq\sqrt{i}) &= \operatorname{ker}' q + i \operatorname{kei}' q. \end{aligned}$$

[9] It is to be noted that this approximation for $K_o(\rho)$ differs from the expression used by J. J. Thomson, "Recent Researches in Electricity and Magnetism," p. 263. Thomson's formula (2) from which his approximation was derived, contains a number of errors and should read

$K_o(x) = (-C + \log 2i - \log x) J_o(x) - 2 J_2(x) - \frac{1}{2} J_4(x) + \frac{1}{3} J_6(x) \ldots \ldots,$
where $C = .5772 \log = \log \gamma$.

$q > 10$

$$J_o(q\sqrt{-i}) = \frac{e^{q/\sqrt{2}}}{\sqrt{\pi q}}\left[\cos\left(\frac{q}{\sqrt{2}} - \frac{\pi}{8}\right) + i\sin\left(\frac{q}{\sqrt{2}} - \frac{\pi}{8}\right)\right]$$

$$J_o'(q\sqrt{-i}) = i\,J_o(q\sqrt{-i})$$

$$K_o(q\sqrt{-i}) = \sqrt{\frac{\pi}{2q}}\,e^{-q/\sqrt{2}}\left[\cos\left(\frac{q}{\sqrt{2}} + \frac{\pi}{8}\right) - i\sin\left(\frac{q}{\sqrt{2}} + \frac{\pi}{8}\right)\right]$$

$$K_o'(q\sqrt{-i}) = -\,i\,K_o(q\sqrt{-i})$$

Note II—Transformation of Fourier-Bessel Expansion

In problems involving Fourier-Bessel expansions it is sometimes necessary to transform quantities of the form

$$\frac{\cos s\phi_j}{\rho_j^s},\ \frac{\sin s\phi_j}{\rho_j^s},\ \log \rho_j,$$

from the system of coordinates ρ_j, ϕ_j to the systems ρ, ϕ or r, θ which are related as shown in Fig. 5.

The necessary formula may be derived as follows. We have

$$\frac{\cos s\phi_j + i\sin s\phi_j}{\rho_j^s} = \frac{e^{is\phi_j}}{\rho_j^s} = \left(\frac{e^{i\phi_j}}{\rho_j}\right) = \frac{1}{Z_j^s},$$

where Z_j is the conjugate of the vector $Z'_j = \rho_j\epsilon^{i\phi_j}$. Similarly we may write

$$Z = \rho\epsilon^{i(\phi-\pi+2a_j)},$$

$$C_j = c_j\,\epsilon^{(\pi+a_j)}$$

The vectors Z_j, Z and C, as may be seen from Fig. 5, have the lengths ρ_j, ρ and c, respectively, and the directions indicated by the arrows.

By vector addition,

$$Z'_j = Z + C_j$$

whence

$$Z_j = Z' + C'_j,$$

where Z' and C'_j are the conjugates of Z and C_j respectively.

By expansion

$$\frac{1}{Z_j^s} = \frac{1}{(Z' + C'_j)^s} = \frac{1}{C_j'^s}\left[1 - \frac{s}{1}\frac{Z'}{C'_j} + \frac{s(s+1)}{1.2}\frac{Z'^2}{C'^2} - \frac{s(s+1)(s+2)}{1.2.3}\frac{Z'^3}{C_j'^3} + - - \right]$$

We have
$$\frac{1}{C_j'^s} = \frac{\epsilon^{is(\pi+a_j)}}{c_j^s} = (-1)^s \frac{\epsilon^{isa_j}}{c_j^s},$$

and
$$\frac{Z'^n}{C_j'^n} = \frac{\rho^n}{c_j^n}\epsilon^{in(2\pi-\phi-a_j)} = \frac{\rho^n}{c_j^n}\epsilon^{-in(\phi+a_j)}$$

Therefore
$$\frac{\epsilon^{si\phi_j}}{\rho_j^s} = \frac{1}{Z_j^s} = \frac{(-1)^s}{c_j^s}\Big[\epsilon^{isa_j} - \frac{s}{1}\frac{\rho}{c_j}\epsilon^{-i(\phi-a_j(s-1))}$$
$$+ \frac{s\,(s+1)}{1.2}\frac{\rho^2}{c_j^2}\epsilon^{-i(2\phi-a_j(s-2))} - \Big]$$

Equating the real and imaginary parts gives

$$\frac{\cos s\phi_j}{\rho_j^s} = \frac{(-1)^s}{c_j^s}\Big[\cos sa_j - \frac{s}{1}\frac{\rho}{c_j}\cos(\phi - a_j[s-1]) +$$
$$\frac{s\,(s+1)}{1.2}\frac{\rho^2}{c_j^2}\cos(2\phi - a_j[s-2]) - - - - - -\Big].$$

$$\frac{\sin s\phi_j}{\rho^s{}_j} = \frac{(-1)^s}{c_j^s}\Big[\sin sa_j + \frac{s}{1}\frac{\rho}{c_j}\sin(\phi - a_j[s-1]) +$$
$$\frac{s\,(s+1)}{1.2}\frac{\rho^2}{c_j^2}\sin(2\,\phi - a_j[s-2]) - - - - - - -\Big].$$

Similarly

$$\log Z_j = \log(C_j' + Z')$$
$$= \log C_j' + \frac{Z'}{C_j'} - \frac{1}{2}\frac{Z'^2}{C_j'^2} + \frac{1}{3}\frac{Z'^3}{C_j'^3} - - - - - - - -$$
$$= \log C_j' + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\frac{\rho^n}{c_j^n}\epsilon^{\,in(\phi-a_j)}.$$

Equating real and imaginary parts we have

$$\log \rho_j = \log c_j + \frac{\rho}{c_j}\cos(\phi - a_j) - \frac{1}{2}\frac{\rho^2}{c_j^2}\cos 2\,(\phi - a_j) + \ldots +,$$
$$\phi_j = \frac{\rho}{c_j}\sin(\phi - a_j) - \frac{1}{2}\frac{\rho^2}{c_j^2}\sin 2\,(\phi - a_j) + \ldots +.$$

The following formulas may be derived in a similar manner:

$$\frac{\cos s\phi_j}{\rho_j^s} = \frac{(-1)^s}{c^s}\Big[1 + \frac{s}{1}\frac{r}{c}\cos(\theta - \gamma_j) + \frac{s(s+1)}{1.2}\frac{r^2}{c^2}\cos 2\,(\theta - \gamma_j) + \ldots +\Big],$$
$$\frac{\sin s\phi_j}{\rho_j^s} = \frac{(-1)^{s+1}}{c^s}\Big[\frac{s}{1}\frac{r}{c}\sin(\theta - \gamma_j) + \frac{s(s+1)}{1.2}\frac{r^2}{c^2}\sin 2\,(\theta - \gamma_j) + \ldots +\Big],$$
$$\log \rho_j = \log c - \frac{r}{c}\cos(\theta - \gamma_j) - \frac{1}{2}\frac{r^2}{c^2}\cos 2\,(\theta - \gamma_j) - - - - - - .$$

NOTE III—DETERMINATION OF q_1, q_2, ETC.

The equations (47),

$$q_n = (-1)^n \lambda_n \frac{\zeta^{2n}}{n} (A + S_{nn} B_o) - (-1)^n \lambda_n \frac{\zeta^{2n}}{n!} \Sigma_n \quad n = 1 - - \infty$$

are linear in the variables $q_1, q_2 -$, since

$$\Sigma_n = n!\, S_{n-1,n+1}\, q_1 - \frac{n!}{1!} S_{n-2,n+2}\, q_2 - - - .$$

The values $q_1, q_2 \ldots$ may be determined by a method of approximations, q_n being the limit of the sequence

$$q_n^{(0)}, q_n^{(1)}, q_n^{(2)} - - - - - - ,$$

the successive terms of which are defined by the expressions

$$q_n^{(0)} = (-1)^n \lambda_n \frac{\zeta^{2n}}{n} (A + S_{nn} B_o),$$

$$- - - - - - - -$$

$$q_n^{(j+1)} = (-1)^n \frac{\zeta^{2n}}{n!} (A + S_{nn} B_o) - (-1)^n \frac{\zeta^{2n}}{n!} \Sigma_n (q^{(j)}),$$

where $\Sigma_n (q^{(j)})$ is the value of Σ_n when $q_1, q_2 -$ are replaced by $q_1^{(j)}, q_2^{(j)} - - -$.

This method, however, while formally simple and direct is not usually well adapted for numerical solution. For all sizes of armor wire and for frequencies of practical importance the argument ζ in the expression (48) is small compared with μ and the quantities,

$$\lambda_1, \lambda_2,$$

are all nearly unity. This suggests the use of the following method of solution of equations (47).

The solution of the auxiliary set of equations

$$p_1 = -\zeta^2 (A + S_{11} B_o) + \frac{\zeta^2}{1!} \Sigma_1(p),$$

$$p_n = (-1)^n \frac{\zeta^{2n}}{n} (A + S_{11} B_o) - (-1)^n \frac{\zeta^{2n}}{n!} \Sigma_n(p),$$

in the auxiliary variables $p_1, p_2 -$ may be written,

$$p_1 = -\zeta^2 C_{11} (A + S_{11} B_o) + \frac{\zeta^4}{2} C_{12} (A + S_{22} B_o) + \ldots + ,$$

$$p_n = -\zeta^2 C_{n1} (A + S_{11} B_o) + \frac{\zeta^4}{2} C_{n2} (A + S_{22} B_o) + \ldots + ,$$

in which C_{11}, etc., are numerics. This solution is effected by retaining a finite number of equations and an equal number of variables and solving by the usual methods. It will be found that except in extreme cases, a very good approximation can be gotten by ignoring all the p's except the first four. The q's may then be obtained by the relation

$$q_n = p_n + d_n$$

d_n being defined by

$$d_n = (\lambda_n - 1)p_n - (-1)^n \lambda_n \frac{\zeta^{2n}}{n!} \Sigma_n(d).$$

This system is easily adapted to solution by successive approximations,

$$d_n = d_n^{(0)} + d_n^{(1)} + d_n^{(2)} + - - -$$

in which

$$d_n^{(0)} = \left(1 - \frac{1}{\lambda_1}\right) C_{n1}p_1 + \ldots + \left(1 - \frac{1}{\gamma_n}\right) C_{nn}p_n,$$

$$d_n^{(j+1)} = \left(1 - \frac{1}{\lambda_1}\right) C_{n1}d_1^{(j)} + \ldots + \left(1 - \frac{1}{\lambda_n}\right) C_{nn}d_n^{(j)},$$

C_{n1}, etc., being the numerical coefficients which appear in the expressions for $p_1, p_2 \ldots$.

A very good approximation which holds in most cases is

$$d_n = (\lambda_1 - 1)\, C_{n1}\, p_1 + (\lambda_2 - 1)\, C_{n2}\, p_2 + \ldots + (\lambda_n - 1) C_{nn} p_n.$$

Analysis of the Energy Distribution in Speech[1]

By I. B. CRANDALL and D. MacKENZIE

SYNOPSIS: *The frequency distribution of energy in speech* has been determined for six speakers, four men and two women, for a 50-syllable sentence of connected speech, and also for a list of 50 disconnected syllables. The speech was received by a condenser transmitter whose voltage output, amplified 3,000 fold, was impressed on the grids of twin single stage amplifiers. The unmodified output of one of these amplifiers was measured by a thermocouple and was a known function of the total energy received by the transmitter, corrections being made for the slight variation with frequency of the response of the circuit. The output of the other amplifier was limited by a series resonant circuit to a narrow band of frequencies, the energy in this band being measured by a second thermocouple. The damping of the resonant circuit was so chosen that sufficient resolving power and sufficient energy, sensitiveness were obtained over the range from 75 to 5,000 cycles per second; and 23 frequency settings were made to cover this range. For each syllable simultaneous readings were recorded on the two thermocouples at each frequency setting. The consecutive syllables were pronounced deliberately by each speaker, maintaining as nearly as possible the normal modulation of the voice. Corrections were applied to offset the unavoidable variations in total energy incidental to repetition of a given syllable. 13,800 observations were made for all speakers. *The energy distribution curves* obtained are essentially the same for connected as for disconnected speech, and indicate that differences between individuals are more important than variations due to the particular test material chosen. A composite curve drawn from the individual curves shows a great concentration of speech energy in the low frequencies, a result which would not be expected from data previously published by others. The actual results contain a factor due to standing waves between the speaker's mouth and the transmitter, a complication always present in telephoning; this could not be eliminated.

The rate of energy output in speech for the normally modulated voice, was determined from the readings for total energy and was found to be about 125 ergs per second.

IN the study of speech and its reproduction by mechanical apparatus it is necessary to consider its composition from several different points of view. We desire first of all to know the actual frequency distribution of the total energy in speech, as well as the separate distributions for each individual sound. We also desire to know the apparent distribution of energy, that is, the distribution as perceived by the ear. Finally, we wish to know the importance of each frequency, that is, the contribution to "articulation" or "quality" in the exact reproduction of speech which can be traced to the energy of each elementary band of frequencies in the speech range. In all three cases certain frequency functions are used to represent these distributions. The advantage of considering these different frequency distribution functions separately has already been indicated by one of the present writers.[2]

[1] Reprinted from THE PHYSICAL REVIEW, N.S., Vol. XIX. No. 3, March, 1922.

[2] "The Composition of Speech," PHYS. REV., X, p. 74, 1917.

In our judgment the most important of these data of speech study is the *actual energy distribution*, considering speech as "a continuous flow of distributed energy," in accordance with the ideas expressed in the earlier paper. The present paper offers a determination of this fundamental factor.

To determine the energy distribution in speech to a high degree of accuracy it would be desirable to analyze a certain amount of connected speech and take a time average of the energy distribution of the whole. This is not feasible at the present time, but a very close approach to this result has been made. The method consists in analyzing the speech waves as impressed on a condense transmitter, using a tuned circuit to transmit narrow frequency bands of energy and pronouncing the separate syllables of the connected speech so slowly that the kick of a direct current galvanometer connected to an A. C. thermocouple can be separately read for each syllable. Using a suitable calibration for the whole apparatus, the magnitude of this kick can be interpreted in terms of the time integral of the energy at a particular frequency setting for each syllable. A mean of the readings for all the syllables in the "speech" at any frequency setting gives the relative energy at that frequency.

The present method is a modification of an earlier method in which approximate analyses of speech sounds were made, using a condenser transmitter, tuned circuit, an amplifying-rectifying circuit, and ballistic galvanometer. The method is, however, much improved as we now have very accurately calibrated condenser transmitters of better design,[3] and a great deal of care has been taken to calibrate the successive elements of the train of apparatus, and increase the resolving power.

Experimental Procedure

Sound waves emitted from the mouth of the speaker are allowed to fall upon the diaphragm of a condenser transmitter, connected in the conventional manner to the input of a three-stage amplifier. The output of this is impressed upon the input circuits of twin single stage amplifiers, potentiometers being interposed to permit regulation of the grid voltages of the twin amplifier tubes.

The output circuits of the fourth stage consist of the high windings of two step down ironclad transformers. These step down transformers have a voltage ratio of 11: 1 and are designed to work between impedances of 6,000 and 50 ohms. The low impedance winding of one of these transformers operates into a thermocouple heater of,

[3] The present design of the condenser transmitter and its calibration are fully treated in a paper by Dr. E. C. Wente which will appear shortly in this Journal.

roughly, 40 ohms resistance. The low side of the other transformer operates through a tuned circuit into a similar thermocouple heater.

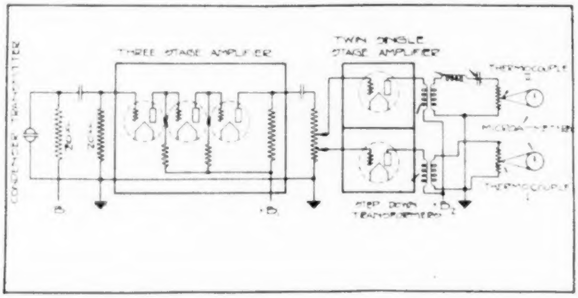


Fig. 1—Circuit Used for the Analysis of Speech. (The Usual Details of the Three-Stage Amplifier Are Not Shown)

The diagram of Fig. 1 exhibits the essential features of the electrical circuits just described.

When the diaphragm of the condenser transmitter is set in vibration by speech a current made up of a range of frequencies flows in the heater of thermocouple I., while the heater of thermocouple II is traversed only by such a band of frequencies as the resonant circuit allows. Fig. 2 shows a number of typical resonance curves obtained

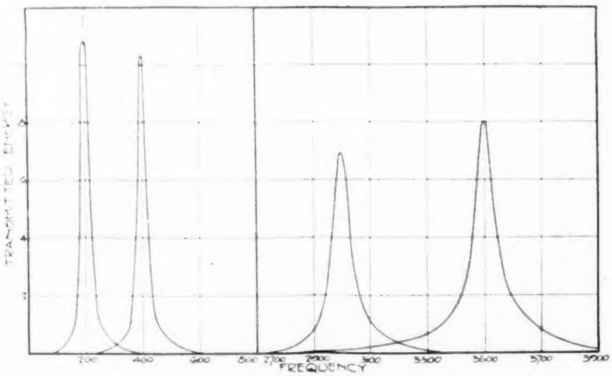


Fig. 2—Resonance Curves Showing the Resolving Power of Apparatus

in the course of calibrating this apparatus. These curves are such that the tuned circuit functions as a filter transmitter only a narrow region of frequencies. One side of the twin amplifier transmits the

entire electrical response of the system; the other side suppresses all save a band of frequencies, the center of this band being shifted by resetting the condenser and inductometer.

Having chosen for analysis a piece of connected discourse, the speaker utters the successive syllables separately but as nearly as may be with the same inflection and volume as if the syllables were continuously spoken. Two observers record the readings of microammeters in the couple circuits of the thermocouples. One of these instruments gives a deflection corresponding to the total energy of the syllable uttered; the deflection of the other instrument corresponds to the energy of the syllable lying within the limits of transmission of the tuned circuit.

Preliminary experiments were carried out to determine the relation between momentary deflection read on the microammeter, and the current momentarily flowing in the thermocouple heater. Currents of different values were caused to flow for intervals of time varying from 0.2 second to 1.2 seconds, and the deflections were found nearly proportional to the product of current squared and time interval; this proportionality was most nearly exact when the current was weak and the time intervals short. For all cases likely to be duplicated in the speech analysis work the error might be taken as about 5 per cent, a quantity small in comparison with the inevitable uncertainties due to other causes.

Quite low damping is attained in the resonant circuit. The values of inductance used ranged from 0.20 to 0.66 henry and the total resistance of the circuit—transformer winding, inductometer coil, thermocouple heater—is of the order of 100 ohms. The damping thus ranges from 75 to 250.

The circuit is calibrated in the following manner:

A switch is so introduced that it is possible to include in series with the thermocouple the resonant circuit, or replace it by a non-inductive resistance whose value is approximately that of the A. C. resistance of the inductometer winding. With the tuned circuit excluded, an alternating current of suitable magnitude is caused to flow in the thermocouple heater; the tuned circuit is then substituted and the new value of the current observed, the input voltage remaining constant. The ratio of current squared "tuned circuit in" to current squared "tuned circuit out" is plotted against frequency, yielding a curve for energy transmission.

Twenty-three bands in all were considered adequate for the analysis of energy distribution in speech; the centers of these were at 75, 100, 200, 300 cycles, 400 to 3,200 cycles by steps of 200; 3,500, 4,000, 4,500,

5,000 cycles per second. Beyond 5,000 cycles per second, the energy is so low as to be impossible of measurement with the apparatus used. A Weston Type 322 microammeter recorded the couple current for the tuned circuit side of the twin single stage amplifier. With this instrument and the thermocouple used, 0.2 microampere in the couple circuit corresponds to one-quarter of a milliampere in the heater, and this is the lowest readable deflection of the Weston instrument.

Reduction of Observations

Three corrections have to be made, the first being the correction for varying volume.

Simultaneous observations are made, at each setting of the tuned circuit, of the filtered and the unfiltered energy of each syllable. It is not possible to utter a given syllable with the same intensity and at the same distance from the transmitter for every one of twenty-

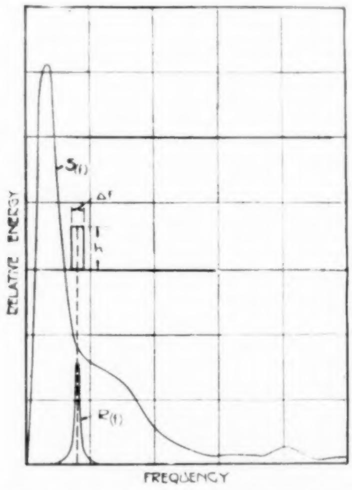


Fig. 3—Illustrating Correction of Observations, Necessary Because of Variation in Resolving Power with Frequency Setting

three times. Accordingly, the "unfiltered" readings are averaged and each of the filtered readings for each syllable reduced from the value actually observed to the value that would have been read had the volume and distance been such as to give the average "unfiltered" reading. This procedure is quite legitimate if it be granted possible to maintain a definite composition of the syllable in question throughout the changes of the tuned circuit setting.

A second correction was made for the varying area of tuned circuit curves.

In Fig. 3 let $S(f)$ be the speech spectrum determined by ideal methods; "R" the transmission curve of the tuned circuit, set for a resonant frequency f. An ideal transmission curve would be a rectangle when plotted in this figure, of height "h" and transmission range Δf.

The true amount of energy $S(f)$ associated with frequency f, and the experimentally determined value which we may call $(\bar{S}f)$ are connected by the relation

$$h\bar{S}(f)\Delta f = \int_f^{f+\Delta f} S(f)R(f)df$$

and if we make

$$h\Delta f = \int_f^{f+\Delta f} R(f)df$$

we may take for all practical purposes $S(f) = \bar{S}(f)$, considering the narrowness of the transmission range. We must therefore find the factor $h\Delta f$, proportional to the area of each tuned circuit curve and divide the energy received through the filtered side by $h\Delta f$, in order to obtain $S(f)$. This treatment may be gone through for each syllable individually, but it is more convenient to sum the tuned circuit readings for all the syllables used, corrected one at a time for varying volume, and then apply the curve area correction to this sum.

A third correction was made for the varying frequency-sensitivity of the whole apparatus. Thus far we have discussed only the electrical energy in the output circuit of the fourth stage. It remains to show in what way this is related to the mechanical energy of the diaphragm, and this in turn to the incident sound energy.

The calibration of the circuit as a whole was made by introducing a small resistance carrying alternating current in series with the condenser transmitter, thus introducing a known potential drop in the undisturbed input mesh of the circuit.

An amplification curve is appended (A, Fig. 4) which gives to an arbitrary scale the ratio of volts output to volts input as a function of frequency, for the system as actually operated. The calibration of the condenser transmitter, shown in Fig. 4, Curve C, gives the open circuit voltage of the transmitter per unit pressure on the diaphragm as a function of frequency. The product of these curves is the volts output per unit alternating pressure on the diaphragm, and the square of this product, curve E is proportional to the electrical energy output per unit sound energy incident on the diaphragm, if we assume that the sound energy is proportional to the square of the alternating pressure. This point, however, requires some further discussion, which will be given later on.

It is plain from curve E that the response of the system is a maximum of frequencies in the neighborhood of 2,250 cycles. If, now, the observations already corrected for varying volume and for area of resonance curves, are subjected to further correction for the exaggeration of these frequencies, it is possible to draw a curve which shall

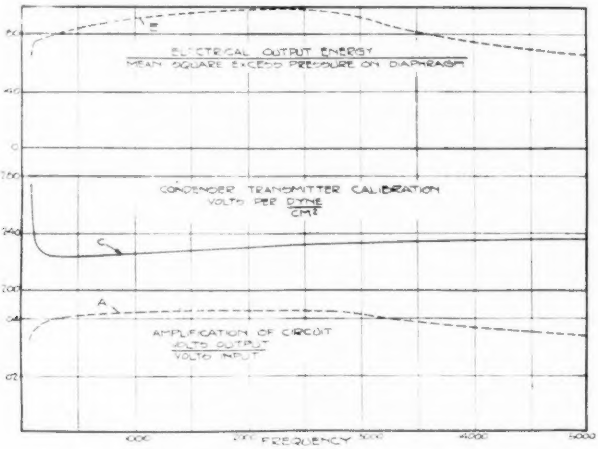


Fig. 4—Energy-Frequency Characteristics of the Apparatus

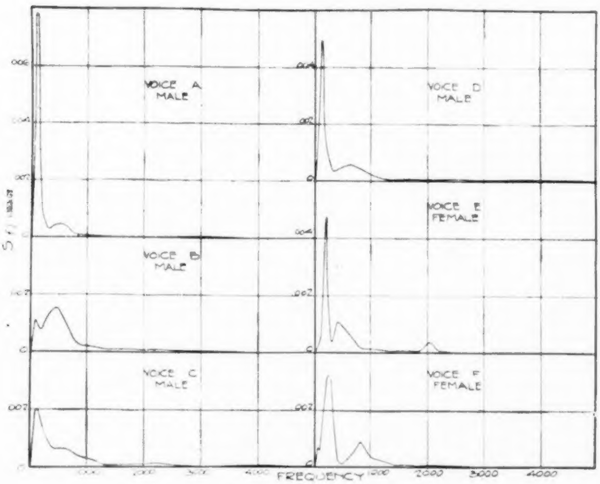


Fig. 5—Analysis of Individual Voices

exhibit the mean square of the excess pressure on the diaphragm, as a function of frequency in the voice exciting the vibration. We obtain this corrected curve by dividing the results, after the first and second corrections above have been made, by the ordinates of curve *E*.

Observations

In order to investigate the possibilities of this method it was decided to work with a rather short piece of connected speech, and to use a limited number of observers, on account of the large number of observations which are required for each separate syllable. With six speakers (four men and two women) each pronouncing the test sentence of fifty syllables for each of the twenty-three frequency settings, 6,900 separate observations were required. It is believed that representative results have been obtained from these observations, but if this is not the case then some method of graphical registration of the energy-time curve of speech for the different frequency settings must be applied in order to handle the vast amount of data involved in work on an appreciably larger scale.

The test sentence used was as follows:

"*Quite* four score and seven years ago our father brought forth on this continent, a *nice* new nation, conceived in liberty, and dedicated to the proposition that all men are created equal."

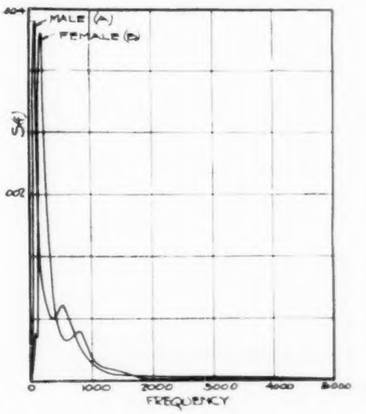


Fig. 6—Energy Distribution: Composite Curves of Male and Female Voices

The two *italicized* words were added to the first sentence of the "Gettysburg Address" in order to bring the total up to fifty syllables, and improve the balance between the vowel sounds.

The resulting speech-energy curves are shown in Figs. 5, 6 and 7,

plotted so that $\int_0^{\infty} S(f)\,df = 1$ in each case. In Fig. 5 the individual curves for each of the six speakers are shown on a small scale; in Fig. 6 the composite curve for the men and the composite curve for the women, drawn separately, and in Fig. 7 the composite curve for all six speakers, giving the data of curves 6*A* and 6*B* equal weight.

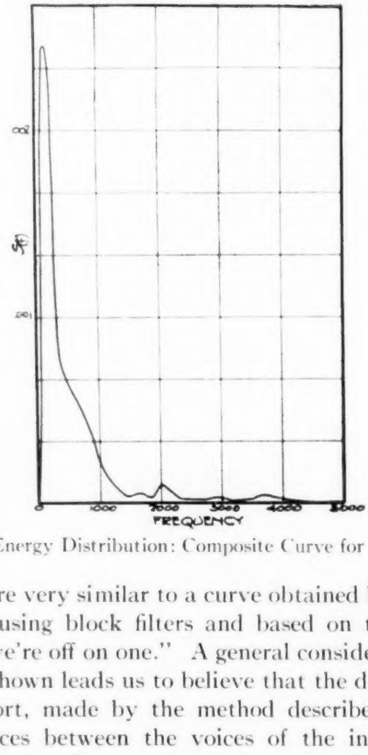


Fig. 7—Energy Distribution: Composite Curve for All Voices

These curves are very similar to a curve obtained by Dr. Fletcher of this laboratory, using block filters and based on the simple calling sentence "Now we're off on one." A general consideration of this fact and of the data shown leads us to believe that the differences between curves of this sort, made by the method described are due rather more to differences between the voices of the individual speakers than to the particular piece of connected speech which is chosen, provided the speech is of reasonable length. The differences between the different voices are so marked that we should expect them to remain even though we used as test material a connected speech ten or fifty times as long as the sentence used.

The Energy Distribution in Speech

An interesting comparison may be made between the curves shown for the energy distribution of "continuous speech" and certain speculative curves previously constructed to indicate the energy distribu-

tion. One of these curves is shown in Fig. 8. Curve *A* was constructed by one of the writers in 1916 in an attempt to synthesize the energy curve from the energy distributions of the vowel sounds, using the vowel analyses of Dr. Dayton C. Miller. Curve *C* is the composite "continuous speech" curve of Fig. 7. The vowel sounds analyzed by

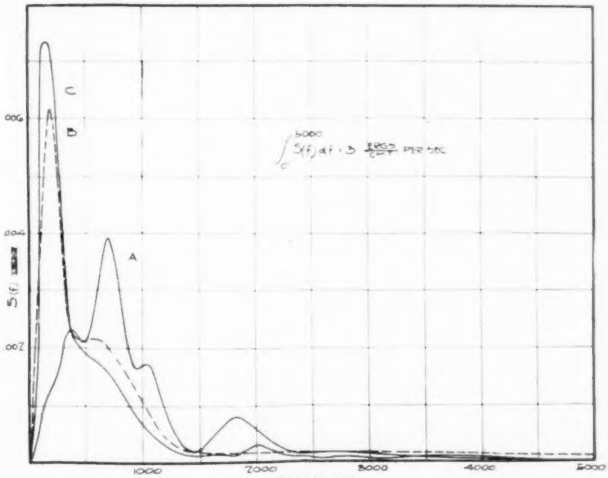


Fig. 8—Energy Distribution: *A*. Synthesized from Vowel Records of D. C. Miller (1916). *B*. Disconnected Speech Analysis of this Paper. *C*. Connected Speech Analysis of this Paper (from Fig. 7)

Miller were intoned and the vowel sounds analyzed by us were spoken, but Miller's work seemed to show that there was no essential difference between intoned and spoken vowel sounds. There is, however, a very noticeable difference between Curve *A* and Curve *C*, the energy in the fundamental tone of the speaker's voice coming out much more strongly in Curve *C*. We should expect that our improved apparatus would record the energy in the lower frequencies more correctly than the apparatus heretofore used but as we used different test material (connected speech instead of disconnected syllables or vowel sounds) it is not immediately evident which of these two factors is responsible for the differences between the *A* and the *C* curves.

In order to investigate this point more fully the testing routine for all six speakers was repeated, using instead of the fifty-syllable sentence, the fifty disconnected syllables of one of the standard articulation testing lists, as used by Dr. Fletcher in this laboratory. The results for energy distribution are shown in Fig. 9, Curve *A* being

the mean energy distribution for the four male speakers, using the syllables, while Curve *B* is the mean energy distribution of the two female speakers. Curves 9*A* and 9*B* may be compared with Curves 6*A* and 6*B* which represent the sentence of continuous speech. The two sets of curves are essentially the same as shown in Fig. 8, *C* and *B*

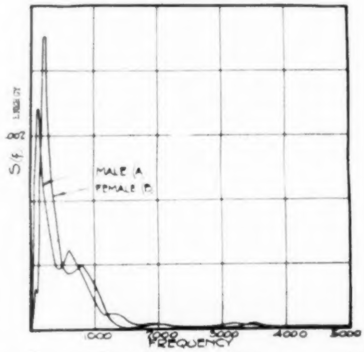


Fig. 9—Energy Distribution in Disconnected Speech

being respectively the composite curves for all speakers, using connected and disconnected speech.

Such small differences as exist between Curves *C* and *B* of Fig. 8 may probably be due to differences in the distribution of the vowel sounds in the connected and disconnected test material. This distribution is given in the following table:

Vowel Sounds	a	ā	á	e	ē	ér	i	ī	o	ō	ó	u	ū	ou	Total
In Sentence	6	6	3	7	3	3	7	2	2	5	3	0	2	1	50
In Syllabic List (No. 174)	4	4	3	4	3	4	3	4	3	4	3	4	4	3	50

Key to Vowel Sounds: a, as in father (or o as in top)
ā, as in tape
á, as in tap
e, as in ten
ē, as in team
ér, as in term
i, as in tip
ī, as in time
o, as in ton
ō, as in tone
ó, as in for
u, as in pull
ū, as in rule
ou, as in house

The similarity between Curves *C* and *B* of Fig. 8 is evidence of the general reliability of the method, and leads to two rather important conclusions.

In the first place, characteristic results have been obtained for a given set of speakers, using two different types of test materials. This

seems to show that the choice of test material does not require especial consideration, provided it is of sufficient length. It seems to be a matter of rather greater importance to increase the number of observers.

In the second place, it seems that for the actual energy distribution, the results previously obtained from the vowel analyses are definitely in error, in that they show relatively little energy associated with the lower voice frequencies.

Criticism of the Results

The foregoing treatment provides a curve showing the frequency distribution of the square of the excess pressure on the diaphragm.

In an undisturbed field of sound energy we have for the intensity

$$I = \frac{P^2}{2\rho a}$$

in which ρ is the mean density of the medium, a the velocity of sound in the medium and P the maximum excess pressure.

It remains for us to consider in how far the results obtained represent the frequency distribution of sound energy in speech.

Due to the fact that at frequencies where the sound wave-length is short and comparable with the diameter of the transmitter, considerable reflection takes place, and the pressure on the diaphragm is proportionately greater for these frequencies than for those which are not accompanied by strong reflection. In this respect again the higher frequencies provoke the greater response in the system.

The following experiment was tried to investigate this variation. A wall six feet square, with a central hole to fit over the condenser transmitter, was brought up to make the transmitter a part of a plane wall. The clearance around the periphery of the transmitter was tightly closed, and reflection was to be expected at all frequencies. Where total reflection takes place, a given quantity of sound energy results in twice the alternating pressure on the diaphragm as when no reflection occurs. That is, the resulting electrical energy observed should be four times as great for total reflection as for no reflection. The wall was expected to cause reflection at all frequencies, and the experiment consisted in reading the electrical response, with and without the wall, the condenser transmitter being exposed to tones of frequencies from 200 to 10,000 cycles per second under definite adjustments of the supply circuit of a receiver producing this tone. When the frequency is low, little reflection takes place from the transmitter standing alone, and bringing up the wall should cause a great increase in the response

of the system. At high frequencies the transmitter should reflect nearly as much alone as when part of a large wall, and the readings with and without the wall should be nearly equal. Plotting ratio of response without, to response with the wall was expected to yield a curve which could be used to make the final reduction of electrical output to incident sound energy, and so permit a more accurate determination of the spectrum of sound energy of the voice.

No consistent results were obtained after several trials and the experiment was abandoned. The failure is doubtless to be ascribed to standing waves, the character of which is very sensitive to the location in the room of the transmitter and the wall. This experiment is to be repeated under more favorable conditions when standing waves can be eliminated.

Thus, the curves finally obtained show no more than the frequency distribution of energy in speech in terms of the mechanical energy of a more or less ideal transmitter diaphragm. However, this information has its value because in any given configuration of transmitter, speaker, and room, there is a definite correspondence between the sound energy of the voice and the force acting on the diaphragm on which it falls, and in telephony at any rate it is this action on the diaphragm with which we are immediately concerned.

In conclusion we may give a determination of the total energy rate of speech, obtained as a by-product of the preceding investigation. Knowing the calibration of the system in absolute units, it is possible to determine the alternating pressure on the condenser transmitter diaphragm exposed to continuous speech from the normally modulated voice under the conditions of the experiment. Using the mean of the values obtained with 9 observers we find for the alternating pressure 11.3 dynes per sq. cm. (r.m.s.) for a distance of 2.5 cm. from mouth to diaphragm. This corresponds to an energy flow of 3.2 ergs per sq. cm. per second. Assuming that this energy flow is distributed uniformly over a hemisphere of 2.5 cm. radius, we may take 125 ergs per second as the total sound energy flow from the lips with the normally modulated voice.

The Nature of Speech and Its Interpretation[1]

By HARVEY FLETCHER

Introduction

VARIOUS phases of this subject have received serious study by phoneticians, otologists, and physicists. On account of its universal interest, it has received attention from men in many branches of science. In spite of the large amount of time devoted to the subject, the progress in understanding its fundamental aspects has been rather slow. At the present time the physical properties which differentiate the various fundamental speech sounds are understood in only a very fragmentary way. Some very interesting and painstaking work has been done on the physical analysis of vowel sounds, but the results to date are far from conclusive. Although several theories have been advanced to explain the way in which the ear interprets sound waves, they are still in the controversial stage.

The material which is presented here is the result of an investigation which has been carried on in the Research Laboratories of the American Telephone and Telegraph Company and Western Electric Company during the past few years.

To make a quantitative study of speech and hearing it is necessary to obtain the speech sounds at varying degrees of loudness and with definitely known amounts of distortion. The main reason why so few real results have been obtained in the investigation of speech sounds is due to the fact that it is extremely difficult to change the volume and distortion of these sounds by acoustic means. Due to recent developments in the electrical transmission of speech it is possible to produce the equivalent of these changes by electrical means. For this purpose a telephone system was constructed which reproduced speech with practically no distortion. It was arranged so that by means of distortionless attenuators the volume of reproduced speech could be varied through a very wide range, and so that by introducing various kinds of electrical apparatus the transmitted speech wave could be distorted in definitely known ways.

A method was developed for measuring quantitatively the ability of the ear to interpret the transmitted speech sounds under different conditions of distortion and loudness. By choosing these conditions properly, considerable information was gained concerning both speech and hearing. This indirect method of attack has a distinct advantage

[1] Presented at a meeting of the Electrical Section of the Franklin Institute held Thursday, March 30, 1922. Reprinted from the Journal of the Franklin Institute for June 1922.

for engineering purposes, in that it measures directly the thing of most interest, namely, the degrading effect upon telephone conversation of introducing electrical distortion into the transmission circuit. However, the application of the results is not limited to this particular field.

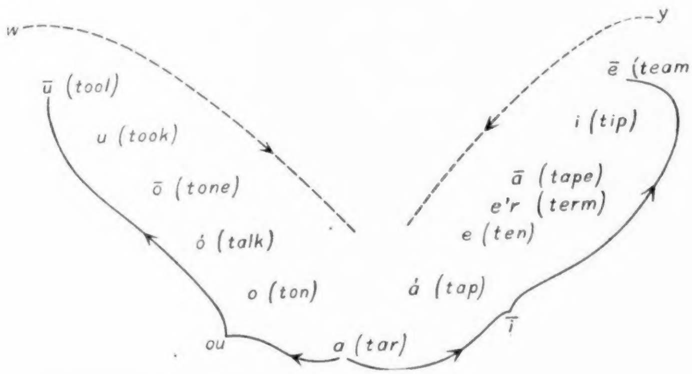
Method of Measuring the Quality of Speech

Briefly stated the method consists in pronouncing detached speech sounds into the transmitting end of the system and having observers write the sounds which they hear at the receiving end. The comparison of the called sounds with those observed shows the number and kinds of errors which are made. The per cent of the total sounds spoken which are correctly received is called the articulation of the system.

Table I.

Classification of the Speech Sounds.

Pure Vowels



Combinational and Transitional Vowels

w — y — ou — ī — h

Semi-vowels

l — r

Stop Consonants

Voiced	Unvoiced	Nasalized	Formation of Stop
b	p	m	lip against lip
d	t	n	tongue against teeth
j	ch	—	tongue against hard palate
g	k	ng	tongue against soft palate

Fricative Consonants

Voiced	Unvoiced	Formation of Air Outlet
v	f	lip to teeth
z	s	teeth to teeth
th (then)	th (thin)	tongue to teeth
zh (azure)	sh	tongue to hard palate

In order to understand the construction of the articulation lists and also to interpret the results of this investigation, I desire to give here a brief classification of the speech sounds, which is based upon the position of the various speech organs when the sounds are being produced. It is shown in the accompanying table (Table I).

The pure vowels are arranged in the vowel triangle, which is familiar to phoneticians. Starting with the sound ū the lips are rounded and there is formed a single resonant cavity in the front part of the mouth. Passing along the left side of the triangle from ū to a the mouth is gradually opened with the tongue lowered to form the successive vowels. Going along the right side of the triangle from a to ē, the tongue is gradually raised to the front part of the mouth forming two resonant chambers in the mouth cavity. An infinite number of different shadings of these vowels may be produced by placing the mouth in the various intermediate positions, but the ones which are shown were chosen as being the most distinct.

The sounds w, y, ou, ī and h are classed as combinational and transitional vowels. As the mouth is placed in the position to say ū and then suddenly changed so as to form any other vowel in the triangle, the result obtained is signified in writing by placing the letter w before the vowel. In a similar way we get the effect usually designated by y if the position of the vowel suddenly changes from ē to any other vowel. An infinite variety of dipthongs can be formed by changing the position of the mouth necessary to form one vowel to that to form another without interrupting the voice. The most distinct and principal ones used in our language are formed by passing from the sound a to either extreme corner of the triangle and are known as ou and ī. When a vowel commences a syllable it is formed by suddenly opening the glottis, permitting the air, which has been held in the lungs, to escape into the mouth, which is formed for the proper vowel. If the glottis remains open and the vowel is started by the sudden contraction of the lungs, we have the effect which is represented in writing by placing an h before the vowel. The sounds l and r are called semi-vowels because the voice train is partially interrupted, although the sound can be continued. The stop and fricative consonants are classified in a manner which is familiar to phoneticians.

It will be noticed that the markings are not those used in the international phonetic alphabet which were entirely too complicated for practical use. Only the bar and accent stroke are used. These can be written quickly and with little chance of error.

In order to pronounce these speech sounds properly, they must

be combined into syllables. For the purpose of this investigation they were combined into mono-syllables of the simple types consonant-vowel, vowel-consonant, and consonant-vowel-consonant.

To eliminate memory effects every possible combination of the sounds into these types of syllables was used unless there was a good reason for excluding it. The complete list contained 8700 syllables. For convenience of testing these syllables were divided into groups of fifty. Each group contained the same kind and number of syllable forms and an equal number of each of the fundamental vowel and consonant sounds.

TABLE II.

Speech-sound Testing List. List No. 160

	Speech-sound	Key-word		Speech-sound	Key-word
1	ha	ho(t)	26	gōb	go+b
2	hā	hay	27	shōl	shoal
3	wā	wa(g)	28	ros	rus(t)
4	wi	wi(th)	29	jod	ju(g)+d
5	vou	vow	30	bok	buck
6	ăr	air	31	zīk	z+(d)ike
7	ez	e(bb)+z	32	bīch	buy+ch
8	ūsh	you+sh	33	kīth	ki(te)+th
9	an	on	34	git	gui(de)+t
10	id	(l)id	35	yif	y+if
11	jouv	jow(l)+v	36	sin	sin
12	moush	mou(nd)+sh	37	tĕrm	term
13	rour	r+our	38	mĕrl	m+earl
14	zūth	z+(s)oothe	39	pĕrv	p+(n)erve
15	hūs	who+s	40	yēt	y+eat
16	chush	ch+(p)ush	41	bēl	b+eel
17	jum	j+(f)oo(t)+m	42	zef	ze(al)+f
18	thup	th+(s)oo(t)+p	43	weng	whe(n)+ng
19	fuch	foo(t)+ch	44	kev	k+ev(er)
20	wông	wa(ll)+ng	45	hăng	hang
21	chôth	cha(lk)+th	46	păg	p+(r)ag
22	tôj	ta(ll)+j	47	yās	y+ace
23	kôg	k+aug(er)	48	dāp	d+ape
24	fōn	(tele)phone	49	yang	ya(cht)+ng
25	dōs	dose	50	lan	l+on

To illustrate the technique of articulation testing a sample list is given in Table II. In the first column the syllable is given in its phonetic form. A key-word showing how each syllable is pronounced is given in the second column. These syllables were written on cards which were shuffled each time before they were used, so that the order in which they were pronounced was entirely haphazard. One hundred and seventy-four similar lists were used in this work. In order to eliminate personal peculiarities, several

callers and observers were used. In Table III are shown the results obtained by an observer when this list was transmitted over a system which eliminated all frequencies above 1250 cycles per second.

TRANSMISSION BRANCH
ARTICULATION TEST RECORDING SHEET

WORD ARTICULATION 40

TITLE OF TEST 320311
CONDITION TESTED Low Pass Filter - 1250~
Attenuation = 5 napiers down

DATE 2-7-20
TEST No. 11
LIST No. 160

OBSERVER M A
CALLER H E D

No.	OBSERVED	CALLED	ERRORS	No.	OBSERVED	CALLED	ERRORS
1	tan	térm	ér - a, m - n	26	zip	thup	th - z, u - i
2	zit	gīt	g - z, ī - i	27	kód	tój	t - k, j - d
3	wa	wá	á - a	28	tish	chush	ch - t, u - i
4	dāp	✓		29	yang	✓	
5	gōb	✓		30	zēt	zūth	ū - ē, th - t
6	yis	yif	f - s	31	ref	ros	o - e, s - f
7	māl	mérl	ér - ā	32	jum	✓	
8	thin	sin	s - th	33	jóg	kóg	k - j
9	zīp	zīk	k - p	34	jad	jod	o - a
10	jouv	✓		35	tūth	hūs	h - t, s - th
11	yāt	yās	s - t	36	id	✓	
12	thou	vou	v - th	37	ha	✓	
13	bīp	bīch	ch - p	38	fōn	✓	
14	háng	✓		39	kóth	chóth	ch - k
15	mīs	moush	ou - ī, sh - s	40	rour	✓	
16	dāch	dōs	ō - ā, s - ch	41	an	✓	
17	kev	✓		42	bok	✓	
18	tig	pág	p - t, á - i	43	yēt	✓	
19	kīs	kīth	th - s	44	ór	ár	á - ó
20	hā	✓		45	yēth	ūsh	y inserted, ū - ē, sh - th
21	weng	✓		46	wóng	✓	
22	dēl	bēl	b - d	47	kōv	pérv	p - k, ér - ō
23	thich	fuch	f - th, u - i	48	zēt	zēf	f - t
24	wif	wi	f inserted	49	lan	✓	
25	ez	✓		50	shōl	✓	

EMB TEA

TABLE III.

The correct word is written opposite all of the syllables which were recorded incorrectly. The errors for each of the fundamental sounds were taken from this original sheet and recorded on an analysis

TABLE IV.

Summary Sheet — Average Errors above 3 %

TRANSMISSION BRANCH

ARTICULATION TEST ANALYSIS SHEET

TITLE OF TEST 320311

CONDITION TESTED Low Pass Filter — 1250 ~ Attenuation — Snapers down

DATE 6-22-20   OBSERVERS Average of 8

LIST Nos.

TEST Nos.   CALLERS 2

SOUNDS CALLED	b	ch	d	f	g	h	j	k	l	m	n	ng	p	r	s	sh	th	t	v	w	y	z	[illegible]	[illegible]	% Error
b			10.3																				4.6		21.2
ch						4.9		8.7							3.2	7.3	4.3	24.7							56.6
d	7.8				11.9		5.3																		35.2
f						3.3									26.0	11.7	19.0						3.9		60.8
g			30.6				3.9																		38.6
h													4.8												13.5
j			19.3		4.0																				29.9
k													3.8					33.7							48.0
l																									3.9
m											2.9														6.8
n										31.7															35.4
ng																									3.6
p		3.0						24.4										22.8							55.0
r									10.4														5.6		18.6
s				6.2												21.5	24.4	7.0							64.9
sh				3.6											27.3		13.0								63.2
th		3.4		8.6											21.1	8.5			7.0			10.5	3.1		70.7
t		7.3				3.4		19.5					6.2												42.4
v																	10.7					15.1			32.0
w																									1.0
y														9.3											18.8
z			6.2				4.2									6.2	8.8		7.9						39.8

No. of times each sound is called

Total number of sounds called

Total number of errors

Consonant Articulation 65.8

Letter Articulation 72.2

Word Articulation 41.2

Summary Sheet — Average Errors above 2 %

TRANSMISSION BRANCH

ARTICULATION TEST ANALYSIS SHEET

TITLE OF TEST 320311

CONDITION TESTED Low Pass Filter — 1250 ~ Attenuation — Snapers down

DATE 6-22-20   OBSERVERS Average of 8

LIST Nos.

TEST Nos.   CALLERS 2

Sounds Called	No. of Times Called	a	ā	á	e	ē	er	i	ī	o	ō	ó	u	ū	ou	% ERROR
a				10.2												13.1
ā																7.2
á		13.8			4.3				2.2			3.8				[illegible]
e			2.4	2.3						10.0						17.5
ē																2.9
ér		3.2	38.8	2.7	4.4					3.6	3.8					60.3
i													5.3			8.4
ī																4.2
o		4.6			24.2											31.9
ō																3.9
ó		5.4								2.2						10.2
u								26.1		4.8						34.6
ū													2.8			11.8
ou																5.6

Total number of sounds called

Total number of errors

Vowel Articulation 83.4

Letter Articulation 72.2

Word Articulation 41.2

sheet as shown in Table IV, for example it will be noticed that p was recorded as k 24.4 per cent, as p 45 per cent, and as t 22.2 per cent of the times called. On the other hand the sound w was only recorded incorrectly 1 per cent of the times called.

For this system the consonant articulation was 65.8 and the vowel articulation 83.4.

Description of the System for Reproducing Speech Sounds

The telephone system used in this investigation is probably more nearly perfect than any other which has yet been built. Its essential elements are a condenser transmitter to receive the speech waves and transform them into the electrical form, an amplifier for magnifying the intensity of the electrical speech currents, an attenuator for controlling the intensity, an equalizing network, and a receiver for delivering the speech to the ear. A schematic arrangement of the circuit is shown in Fig. 1.

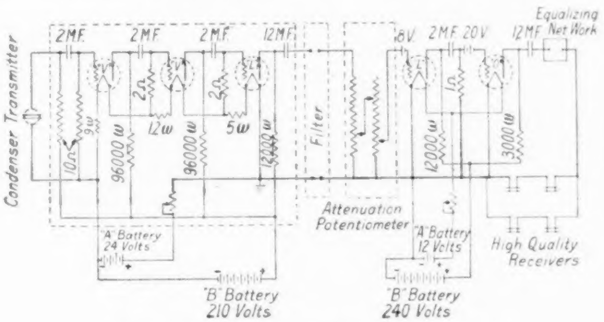


Fig. 1.—High Quality Telephone System

A detailed description of the construction and operation of the condenser transmitter has been given by Crandall and Wente and published in the *Physical Review*.[1] It is simply an air condenser, one of its plates being a flexible metal diaphragm.

A five-stage vacuum tube amplifier was used. Particular care was taken in coupling the stages together, so that the amplifier was practically free from frequency distortion.

The attenuator consisted of a potentiometer arrangement which could reduce the amplitude of the speech waves to approximately one-millionth of their maximum values.

The equalizing network was an arrangement of resistances, con-

[1] Crandall, *Phys. Rev.*, June, 1918; Wente, *Phys. Rev.*, July, 1917.

densers and inductance coils having a frequency selectivity which was the complement of that of the rest of the system.

The telephone receiver was a bipolar type having a special construction which was designed to broaden the range of frequency response.

The reproducing efficiency of the system from the mouth of the speaker to the ear of the listener for each frequency is shown in Fig. 2.

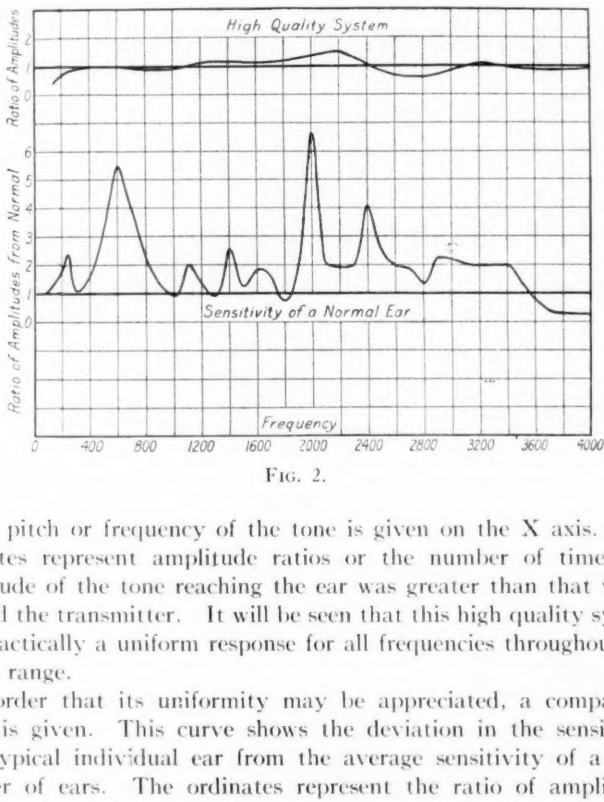


Fig. 2.

The pitch or frequency of the tone is given on the X axis. The ordinates represent amplitude ratios or the number of times the amplitude of the tone reaching the ear was greater than that which entered the transmitter. It will be seen that this high quality system has practically a uniform response for all frequencies throughout the speech range.

In order that its uniformity may be appreciated, a comparison curve is given. This curve shows the deviation in the sensitivity of a typical individual ear from the average sensitivity of a large number of ears. The ordinates represent the ratio of amplitudes at the various pitches which was necessary to bring the tone to the threshold of audibility. It is evident that this deviation is much larger than the departure of the high quality circuit from uniformity.

To show that this particular individual's curve is typical, the curves for both ears of 20 women are given in Fig 3. For convenience these curves are plotted on logarithmic paper. If an arithmetic

scale is used, all of the curves below the mean are crowded together in the small space between zero and one, and all those above the mean are stretched out from one to infinity. By using a logarithmic plot a symmetrical distribution is obtained. The method of obtain-

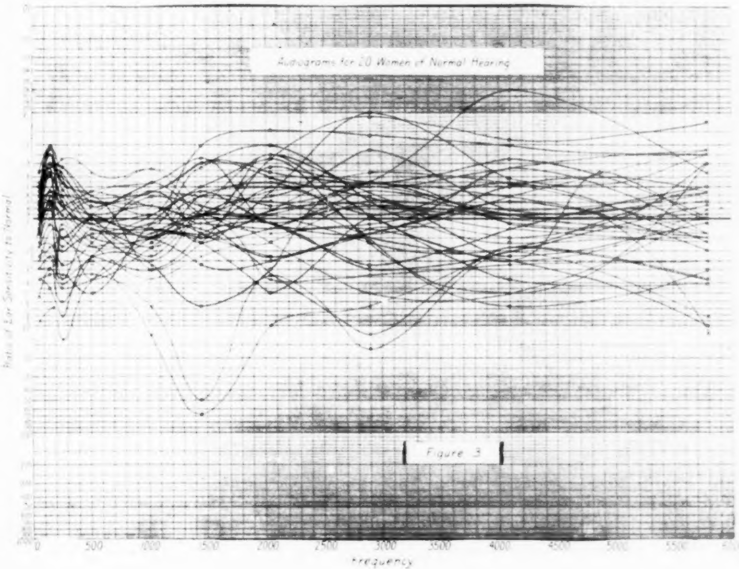


FIG. 3.

ing these ear sensitivity curves was fully described in a recent paper[2] given before the Natural Academy of Sciences.

It is interesting to note that they indicate that each individual has a hearing characteristic which is quite different from other individuals. Consequently speech sounds differently to different persons. Any distortions of the speech sounds will necessarily affect some persons differently from others. It is evident then that in discussing speech and hearing we must deal with statistical averages.

Experimental articulation tests showed that the ear interpreted the speech which was transmitted over this high quality system practically as well as that transmitted through the air. Some may wonder why such good quality is not furnished telephone users in commercial practice: Scientifically speaking, it is possible to furnish such quality, but it is evident that the equipment involved is so com-

[2] Fletcher and Wegel, *Proc. Nat. Acad. Science*, Vol. 8, No. 1, pp. 5–6, Jan., 1922.

plicated that such service would be altogether too costly for commercial use; people could not afford to pay for it.

The Relation Between the Volume and Articulation of Undistorted Speech

Articulation tests were made upon the high quality telephone system described above when it was set to deliver various intensities from the threshold of audibility to very large values. The results shown as syllable articulation values are given by the curve

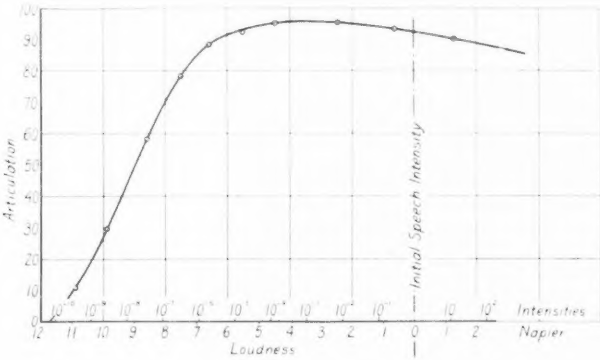


Fig. 4.

in Fig. 4. The abscissas in this curve represent loudness and are expressed as the natural logarithm of the number of times the speech wave amplitude has been decreased from the initial intensity at ½ inch in front of the mouth of the callers. This unit of loudness has never been given a name, and as a matter of convenience in this work it is called a napier. It will be noticed that when the volume is reduced 11½ napiers below the initial speech intensity the articulation becomes zero. This point also represents the value at which the speech becomes inaudible and corresponds to approximately 1/1000 dynes per square centimetre pressure variation against the ear drum. In energy units it is a reduction of ten billion times below the initial speech intensity. For very loud initial speech this point is shifted about 1 napier. For purposes of comparison the intensity reductions are also indicated on the loudness axis.

At 3 napiers below or at about 1/1000 of the initial speech intensity the articulation becomes a maximum. Louder speech than this seems to deaden the nerves so that a person makes a less accurate

interpretation of the received speech. These results were obtained in a room which was especially constructed to exclude outside noise. When noise is present at the receiving station the optimum loudness increases as the noise increases.

The articulation data were analyzed so as to show the errors of each of the fundamental sounds. The curves given in Fig. 5 show the results of this analysis. It will be noticed that the volume at which errors begin to be appreciable is different for the different sounds and is usually higher for the consonants than for the vowels.

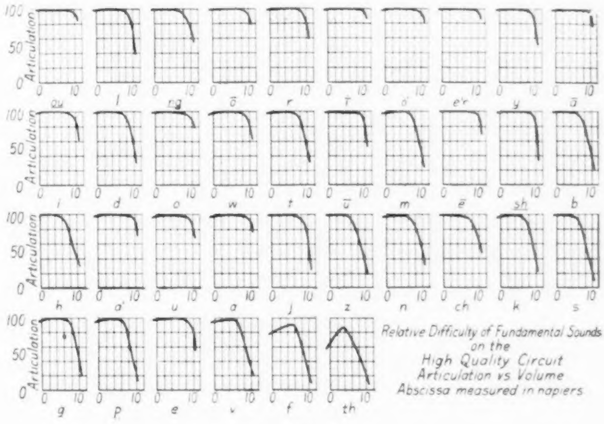


FIG. 5.

Within the precision of the test the intersection point on the X axis was the same for all the sounds, namely at 11.5 napiers.

It will be noticed that the consonants are usually harder to hear than the vowels. However, the speech sounds e and l, r, ng form notable exceptions to this general rule, since the former is among the most difficult, while the latter are among the very easiest speech sounds. The order in which the speech sounds are given here represents their relative difficulty of interpretation when received at average intensities. At all intensities, the sounds th, f and v are the most difficult. Z, h and s become very difficult at weak volumes. The sounds i, ou, er and ó are missed less than 10 per cent of the time, even with "very weak" intensity. At "average" volumes there are only three sounds more difficult than e while at "very weak" volumes there are 23 sounds more difficult. At very weak volumes l, which is the easiest sound at "average" volumes is missed three times as often as e.

We will now pass to a consideration of the effect of distortion upon the articulation of the sounds.

Description of Electrical Filters Used to Produce Distortion

In order to investigate distortion we would like to be able to take the train of speech waves going from the mouth to the ear and operate upon it in various ways such as eliminating frequencies in certain regions without marring or disturbing other frequencies. For example, if all frequencies above 1000 were eliminated, it would be possible to determine what intelligibility is carried by this range of frequencies.

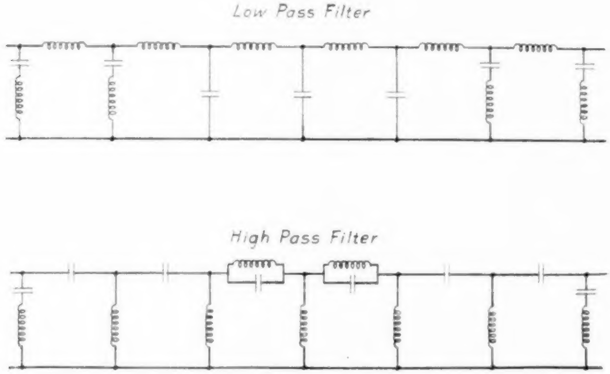


Fig. 6.

Fortunately one of the recent electrical inventions is admirably adapted for this purpose, namely, the electrical wave filter invented by Dr. G. A. Campbell. This device was used extensively in this investigation.

The schematic circuit diagrams of the two types of filters which were used are given in Fig. 6.

This arrangement of coils and condensers produces an electrical conductor with the unusual properties that it transmits without appreciable diminution in amplitude any frequency between certain limits and reduces the amplitude of all frequencies outside these limits to less than 1/1000 of their original value. By varying the numerical values of the inductances and capacities this transmitted range can be placed at any desired position. In the arrangement which was used in the investigation these coils and condensers were

housed in two boxes. The switching mechanism was arranged so that by turning a dial the condensers and coils were connected in such a way that the filter transmitted different frequency bands.

In Fig. 7 are shown the transmission properties of the low pass filter when the dial is set to transmit frequencies from 0 to 1500. It is seen that for frequencies below 1400 the amplitudes of the transmitted tones are always greater than .8 of their initial values, while for frequencies above 1500 the amplitudes are decreased to less than .001 of their initial values. These electrical filters were connected into the high quality circuit between the third and fourth stages

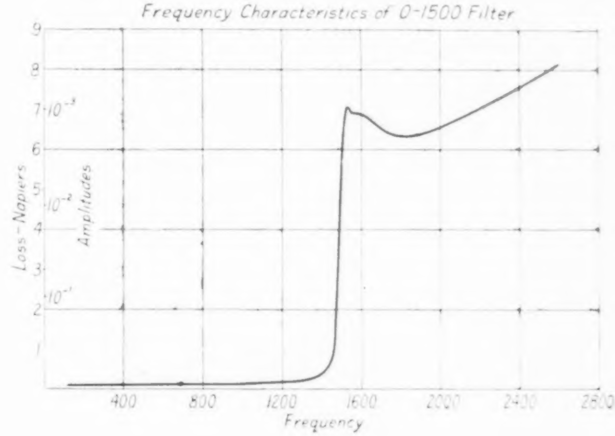


Fig. 7.

of the amplifier as indicated in Fig. 1. This combination formed a system which would pick up a complex sound wave and transmit faithfully to the ear those component frequencies in any desired region and eliminate all other frequencies.

Results of Articulation Tests with Filter Systems

Articulation tests were made with these filter systems and the results analyzed as described above. In Fig. 8 the syllable articulation results are shown in graphical form. The ordinates for the solid curves represent the per cent of the articulation syllables called into the system which were correctly recorded at the observing end. The abscissas represent the so-called "cut off" frequency of the filter. For example on the curve labelled "Articulation L" the point (1000, 40) means that a system which transmits only frequencies

below 1000 cycles per second has a syllable articulation of 40 per cent. Similarly on the curve labelled "Articulation H" the point (1000, 86) means that a system which transmits only frequencies above 1000 cycles per second has a syllable articulation of 86 per cent. The dotted curves show the per cent of the total speech energy which is transmitted through the filter systems used in the articulation tests. These curves are derived from the results of Crandall and MacKenzie which were recently published.[3]

It will be seen that although the fundamental cord tones with their first few harmonies carry a large portion of the speech energy,

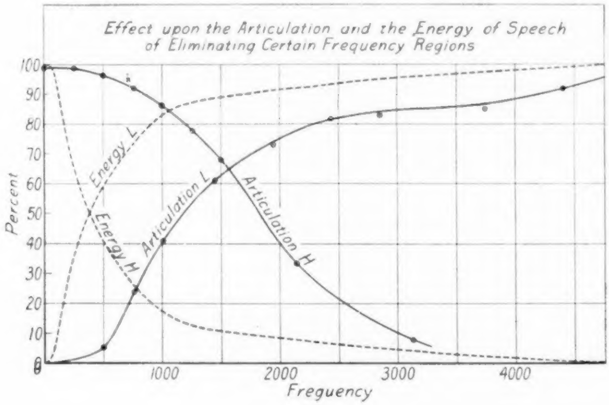


FIG. 8.

they carry practically none of the speech articulation. A filter system which eliminates all frequencies below 500 cycles per second eliminates 60 per cent of the energy in speech, but only reduces the articulation 2 per cent. A system which eliminates frequencies above 1500 cycles per second eliminates only 10 per cent of the speech energy, but reduces the articulation 35 per cent. A system which eliminates all frequencies above 3000 cycles per second has as low a value for the articulation as one which eliminates all frequencies below 1000 cycles per second. This last statement may appear rather astonishing since it is contrary to the popular notion of the relative importance of various voice frequencies from an interpretation standpoint.

The two solid curves intersect on the 1550 cycle abscissa and at 65 per cent articulation, which shows that using only frequencies

[3] See preceding paper.

above or frequencies below 1550 cycles an articulation of 65 per cent will be obtained. The two dotted curves necessarily intersect at 50 per cent.

The curves in Fig. 9 show how the articulation of some of the fundamental speech sounds was affected by eliminating certain frequency regions. The ordinate gives the number of times the sound was written correctly per 100 times called. As in Fig. 8 the

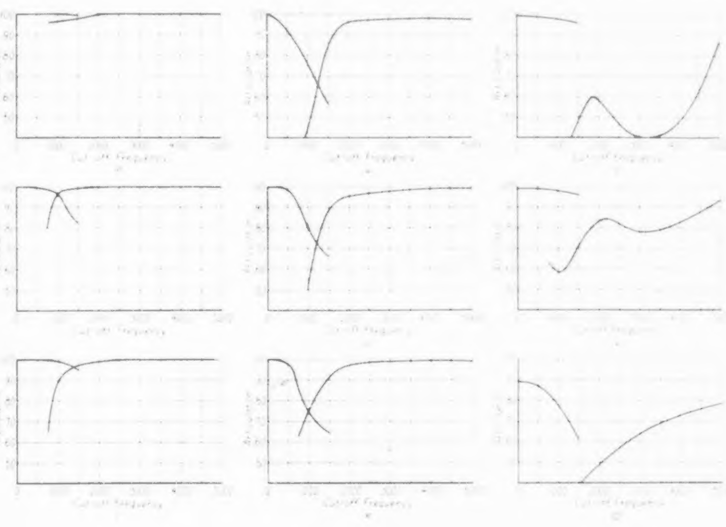


FIG. 9.

left hand curve shows the effect of eliminating all frequencies below and the right hand curve the effect of eliminating all frequencies above the frequency specified by the abscissa.

These nine speech sounds were chosen as representing three important classes. It is seen that the long vowels ē, I and ī can be transmitted with an error of less than 3 per cent when using either half of the range of frequencies. When using either frequencies from 0 to 1700 or from 1700 to infinity ē was interpreted correctly 98 per cent of the time. Similarly I was interpreted correctly 97 per cent of the time when using either the range from 0 to 1000 or 1000 to infinity, and ī 96 per cent of the time when using either the range from 0 to 1350 or from 1350 to infinity. The short vowels, u, o and e are seen to have important characteristics carried by frequencies below 1000. More than a 20 per cent error is made on any of these

three sounds when frequencies below 1000 are eliminated. The elimination of frequencies above 2000 produces almost no effect.

The fricative consonants s, z and th are seen to be affected very differently from those in the other two classes. These sounds are very definitely affected when frequencies above 5000 are eliminated. The sounds s and z are not affected by the elimination frequencies below 1500. It is principally due to these three sounds that the syllable articulation is reduced from 98 per cent to 82 per cent when frequencies above 2500 cycles are eliminated.

A more detailed analysis of the articulation results on all the speech sounds showing the kind as well as the number of errors will be given in a future paper.

Conclusion

In conclusion then we see that the intensity of undistorted speech which is received by the ear can be varied from 100 times greater to one-millionth less than the initial speech intensity without noticeably affecting its interpretation. The intensity must be reduced to one-ten-billionth of that initial speech intensity to reach the threshold of audibility for the average ear. Also it is seen that any apparatus designed to reproduce speech and preserve all of its characteristic qualities must transmit frequencies from 100 to above 5000 cycles with approximately the same efficiency. Although most of the energy in speech is carried by frequencies below 1000, the essential characteristics which determine its interpretation are carried mostly by frequencies above 1000 cycles. In ordinary conversation the sounds th, f and v are the most difficult to hear and are responsible for 50 per cent of the mistakes of interpretation. The characteristics of these sounds are carried principally by the very high frequencies.

It is evident that progress in the knowledge of speech and hearing has a great human interest. It will greatly aid the linguists, the actors, and the medical specialists. It may lead to improved devices which will alleviate the handicaps of deaf and dumb persons. Furthermore this knowledge will be of great importance to the telephone engineer, and since the telephone is so universally used, any improvement in its quality will be for the public good.

These humanitarian and utilitarian motives as well as the pure scientific interest have already attracted a number of scientists to this field. Now that new and powerful tools are available, it is expected that in the near future more will be led to pursue research along those lines.

The Contributors to this Issue

William Wilson, Victoria University of Manchester, 1904–10; M.Sc., 1908; Cavendish Laboratory, Cambridge University, 1910–12, B.A., 1912; Lecturer in Physics, Toronto University, 1912–14; D.Sc. Manchester, 1913. Engineering Department Western Electric Company, 1914– . Dr. Wilson has published numerous papers on radio activity and thermionics and since 1917 has been in direct charge of vacuum tube design.

George A. Campbell, B.S., Massachusetts Institute of Technology, 1891; A.B., Harvard, 1892; Ph.D., 1901; Göttingen, Vienna and Paris, 1893–96. Mechanical Department, American Bell Telephone Company, 1897; Engineering Department, American Telephone and Telegraph Company, 1903-1919; Department of Development and Research, 1919—; Research Engineer, 1908—. Dr. Campbell has published papers on loading and the theory of electric circuits and is also well-known to telephone engineers for his contributions to repeater and substation circuits. The electric filter which is one of his inventions plays a fundamental role in telephone repeater, carrier current and radio systems.

H. M. Trueblood, B.S., Earlham, 1902; Haverford, 1903; Massachusetts Institute of Technology, 1908–09; Ph.D., Harvard, 1913; aid and assistant United States Coast and Geodetic Survey, 1903–08; assistant in physics, Harvard, 1912–14; Joule-Thomson effect in super-heated steam; instructor and assistant professor electrical engineering, University of Pennsylvania, 1914–17; Department of Development and Research, American Telephone and Telegraph Comcompany, 1917—; work on inductive interference.

J. J. Pilliod, E.E., Ohio Northern University, 1908; American Telephone and Telegraph Company, Toledo Home Telephone Company, and Union Switch and Signal Company, short periods, 1904–08; American Telephone and Telegraph Company, Long Lines Department, 1908–11; Engineering Department, 1912–13; Division Plant Engineer, Long Lines Department, 1914–17; Engineer of Transmission, 1918–19; Engineer, 1920–. As Engineer of the Long Lines Department, Mr. Pilliod has been in general charge of engineering work involved in the planning and installation of the newer sections of the cable project described.

John R. Carson, B.S., Princeton, 1907; E.E., 1909; M.S., 1912; Research Department, Westinghouse Electric and Manufacturing

Company, 1910–12; instructor of physics and electrical engineering, Princeton, 1912–14; American Telephone and Telegraph Company, Engeneering Department, 1914–15; Patent Department, 1916–17; Engineering Department, 1918; Department of Development and Research, 1919–. Mr. Carson's work has been along theoretical lines and he has published several papers on theory of electric circuits and electric wave propagation.

J. J. Gilbert, A.B., University of Pennsylvania, 1909; Harvard, 1910–11; Chicago, 1911–12; E.E., Armour Institute, 1915; instructor of electrical engineering, Armour, 1912–17; Captain Signal Corps, 1917–19; Engineering Department, Western Electric Company, 1919, since when he has worked on submarine cable problems.

I. B. Crandall, A.B., Wisconsin, 1909; A.M., Princeton, 1910; Ph.D., 1916; Professor of Physics and Chemistry, Chekiang Provincial College, 1911–12; Engineering Department, Western Electric Company, 1913–. Dr. Crandall has published papers on infra-red optical properties, condenser transmitter, thermophone, etc. More recently he has been associated with studies on the nature and analysis of speech which have been in progress in the Laboratory.

Donald MacKenzie, A.B., Johns Hopkins, 1908; A.M., 1911; Ph.D., 1914; assistant astronomy, 1914–17; associate physicist, Bureau of Standards, 1918–20; Engineering Department, Western Electric Company, 1920–.

Harvey Fletcher, B.S., Brigham Young, 1907; Ph.D., Chicago, 1911; instructor of physics, Brigham Young, 1907–08; Chicago, 1909–10; Professor, Brigham Young, 1911–16; Engineering Department, Western Electric Company, 1916–. The present paper by Dr. Fletcher gives some of the results of an investigation which is being made of the relation between the frequency characteristics of telephone circuits and the intelligibility of transmitted speech. Dr. Fletcher has also published on Brownian movements, ionization and electronics.